Balanced trade as a special case in open economy models

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This note shows how, in a canonical open economy model (the sticky price model of Galí and Monacelli (2005) is one example), one can get balanced trade at all times. We first review relevant model equations under producer currency pricing:

Marginal utility:	$\Lambda_t = C_t^{-\sigma}$
International risk sharing:	$\Lambda_t \mathcal{S}_t = \Lambda_t^*$
Nominal trade balance:	$TB_t^n = P_{H,t} C_{H,t}^* - P_{F,t} C_{F,t}$
Export demand:	$C_{H,t}^* = (1-\alpha) \left(\frac{P_{H,t}}{\mathcal{E}_t P_t^*} \right)^{-\eta} C_t^*$
Import demand:	$C_{F,t} = (1-\alpha) \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t$
Law of one price:	$P_{F,t} = \mathcal{E}_t P_t^*$
Real exchange rate:	$\mathcal{S}_t = \frac{\mathcal{E}_t P_t^*}{P_t}$

These are all the equations we need. However, note for future reference that domestic CPI is $P_t = \left[\alpha P_{H,t}^{1-\eta} + (1-\alpha) P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$, and that we define the terms of trade as $\mathcal{T}_t = \frac{P_{F,t}}{P_{H,t}}$, i.e. as the price of imports in terms of exports.

Substitute the optimality conditions with respect to exports and imports into the net exports definition, and divide by P_t in order to express trade in real terms:

$$\begin{aligned} TB_t^r &= \frac{P_{H,t}}{P_t} (1-\alpha) \left(\frac{P_{H,t}}{\mathcal{E}_t P_t^*} \right)^{-\eta} C_t^* - \frac{P_{F,t}}{P_t} (1-\alpha) \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \\ &= (1-\alpha) \left[\left(\frac{P_{H,t}}{P_t} \right)^{1-\eta} \mathcal{S}_t^{\eta} C_t^* - \left(\frac{P_{F,t}}{P_t} \right)^{1-\eta} C_t \right] \\ &= \frac{1-\alpha}{\alpha + (1-\alpha) \mathcal{T}_t^{1-\eta}} \left[\mathcal{S}_t^{\eta} C_t^* - \mathcal{T}_t^{1-\eta} C_t \right] \end{aligned}$$

Nasty expression, right? Well, life becomes simpler if one makes some (strong) assumptions. Suppose $\eta \to 1$. Now the real trade balance collapses to

$$TB_t^r = (1-\alpha) \left(\mathcal{S}_t C_t^* - C_t \right).$$

If also $\sigma \to 1$, then the risk sharing condition simplifies to $\mathcal{S}_t C_t^* = C_t$, and we get balanced trade $\forall\ t$. It is straight forward to show that the trade balance becomes identical if instead trade is based on local currency pricing. Galí and Monacelli (2005) use the calibration $\eta = \sigma = 1$ in order to derive a quadratic welfare loss function from first principles. However, the underlying assumptions are i) perfect risk sharing internationally, ii) a unitary elasticity of substitution between imports and domestic goods, and iii) an intertemporal elasticity of substitution equal to the relative risk aversion equal to one. Those are strong assumptions!