Inflation Targeting in a Small Open Economy with Sticky Wages and Prices

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Abstract

The New Open Economy Macroeconomics (NOEM) literature has found that domestic price inflation should be a key target for the central banks. Still, these institutions typically pay attention to a broader set of macroeconomic variables, including wages and imported prices. In this paper I combine the sticky wage environment introduced by Erceg, Henderson, and Levin (2000) with the small open economy setup popularized in Galí and Monacelli (2005) to rationalize the observed behavior of the central banks. I derive a second-order approximation to household's welfare losses and evaluate a number of different monetary rules. A simple Taylor rule containing a weighted composite of domestic prices and wages, with the weights being determined by deep parameters, can approximate the optimal policy reasonably well.

1 Introduction

A well known result in the NOEM literature is that domestic price inflation should be an important target for inflation targeting central banks. Still, these institutions typically pay attention to a broader set of macroeconomic variables, including wages and imported prices. In this paper I combine the sticky wage environment introduced by Erceg et al. (2000) (EHL henceforth) with the small open economy setup popularized in Galí and Monacelli (2005) (GM henceforth) to rationalize the observed behavior of the central banks.[1]

Section 2 presents the model and arrives at a New Keynesian wage Phillips curve and the standard New Keynesian price Phillips curve, both augmented with the real wage gap (from flexible equilibrium). Section 3 conducts the policy analysis. I first look at the role of fiscal policy. An interesting observation is that policymakers in a small open economy could find it optimal to tax wage payments instead of subsidizing them. This is in contrast to what is found in closed economy models. Then I establish a second-order approximation to household's welfare losses and evaluate a number of different monetary rules. It is seen that a simple Taylor rule targeting a weighted composite of domestic prices and wages can approximate the optimal policy quite well. The weight on each inflation measure is determined by the relative rigidity and some curvature parameters. Findings are supplemented with an impulse response analysis. Section 4 concludes.

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[1]See Corsetti, Dedola, and Leduc (2010) for an analysis of monetary policy in large open economies.

2 THE MODEL

2.1 HOUSEHOLDS

Consider a small open economy (labeled the home economy) with a continuum of symmetric households indexed by $h \in [0,1]$. A typical household seeks to maximize lifetime utility given by:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u \left(C_{ht|t-k}, N_{ht|t-k} \right) \right\} \tag{1}$$

$C_{ht|t-k}$ is a consumption index of home and foreign goods for a household that was last able to reoptimize its wage k periods ago. $N_{ht|t-k}$ is the household's total labor hours. Utility is isoelastic:

$$u \left(C_{ht|t-k}, N_{ht|t-k} \right) \equiv \frac{C_{ht|t-k}^{1-\sigma}}{1-\sigma} - \frac{N_{ht|t-k}^{1+\varphi}}{1+\varphi} \tag{2}$$

I assume the existence of a complete set of tradable Arrow securities. This makes consumption independent of the wage history, i.e. $C_{ht|t-k} = C_{ht|t} \equiv C_{ht}$. Because households are symmetric and of measure one, household h consumption is also economy wide consumption. I therefore drop the h subscript whenever possible from now on. Consumption yields utility for a representative domestic household according to a nested CES structure:

$$\begin{aligned} C_t &\equiv \left[(1-\vartheta)^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \vartheta^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad & C_{Ht} &\equiv \left(\int_0^1 C_{Hit} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}, \\ C_{Ft} &\equiv \left(\int_0^1 C_{jt} dj \right)^{\frac{\gamma}{\gamma-1}}, & C_{jt} &\equiv \left(\int_0^1 C_{jit} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \end{aligned} \tag{3}$$

The parameter ϑ measures the degree of openness in the economy while η is the substitutability between domestic and foreign goods. C_{Ht} is a CES index of domestically produced goods, represented by the unit interval $i \in [0,1]$. The parameter ϵ_p represents the elasticity of substitution between varieties produced within any given country j, including the home country. C_{Ft} indexes country level imports and γ is the elasticity of substitution for demand towards different countries. Finally, C_{jt} is an index of the different goods $i \in [0,1]$ imported from country j. Equations (1)-(3) characterize preferences of a representative household. Household h faces a sequence of budget constraints:

$$\begin{aligned} \int_0^1 P_{Hit} C_{Hit} di + \int_0^1 \int_0^1 P_{jit} C_{jit} di dj + \int_{\varsigma \in \Omega} V_{t,t+1} (\varsigma) D_{t+1} (\varsigma) d\varsigma \\ = D_t + \int_0^1 W_{ht} N_{iht|t-k} di + T_t \end{aligned} \tag{4}$$

The price on domestic good i is denoted P_{Hit} while the price on good i imported from country j is denoted P_{jit}. $V_{t,t+1}$ is the period t price of an Arrow security yielding a payoff $D_{t+1}(\varsigma)$ if state $\varsigma \in \Omega$ occurs, and zero otherwise. $\int_{\varsigma \in \Omega} V_{t,t+1} (\varsigma) D_{t+1} (\varsigma) d\varsigma$ is the market price of a fully hedged one-period portfolio. Denote $\xi_{t,t+1}$ as the probability that a given state of nature is realized in period $t+1$. Then the effective price of the portfolio can be written as $E_t \frac{V_{t,t+1}}{\xi_{t,t+1}} D_{t+1}$, with D_{t+1} being the (certain) payoff of the portfolio. The stochastic discount factor is thus defined as $Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}}$.

2.1.1 OPTIMAL CONSUMPTION DEMAND

The representative household's optimization problem can be dealt with in several stages. First, for any given level of consumption expenditures on home goods, one must decide how much to buy of each. An equivalent decision has to be made about imported goods from each of the foreign countries. Second, given an expenditures level on imported goods, one must decide import shares from each foreign country. Third, given total consumption expenditures, one must decide the share of home goods relative to imported goods. The result of all these optimization exercises is an aggregate price index and an optimal demand schedule for every specific consumption unit at each stage in the nested system:

$$C_{Hit} = \left(\frac{P_{Hit}}{P_{Ht}}\right)^{-\epsilon_p} C_{Ht}, \quad P_{Ht} = \left(\int_0^1 P_{Hit}^{1-\epsilon_p} di\right)^{\frac{1}{1-\epsilon_p}} \tag{5}$$

$$C_{jit} = \left(\frac{P_{jit}}{P_{jt}}\right)^{-\epsilon_p} C_{jt}, \quad P_{jt} = \left(\int_0^1 P_{jit}^{1-\epsilon_p} di\right)^{\frac{1}{1-\epsilon_p}} \tag{6}$$

$$C_{jt} = \left(\frac{P_{jt}}{P_{Ft}}\right)^{-\gamma} C_{Ft}, \quad P_{Ft} = \left(\int_0^1 P_{jt}^{1-\gamma} dj\right)^{\frac{1}{1-\gamma}} \tag{7}$$

$$\begin{gathered} C_{Ht} = (1-\vartheta)\left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t, C_{Ft} = \vartheta\left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t, \\ P_t = \left[(1-\vartheta) P_{Ht}^{1-\eta} + \vartheta P_{Ft}^{1-\eta}\right]^{\frac{1}{1-\eta}} \end{gathered} \tag{8}$$

Optimal demand for home good i is determined by (5). Optimal demand for good i imported from country j is given by (6). Optimal basket of import consumption from country j is given by (7). Finally, the optimal demand for home and foreign goods are determined by (8).

2.1.2 THE EULER EQUATION

Given the market equilibrium for the aggregators in the previous section, total consumption expenditures are found from (5)-(8) as:

$$\int_0^1 P_{Hit} C_{Hit} di + \int_0^1 \int_0^1 P_{jit} C_{jit} didj = P_t C_t$$

Thus, the problem for the household with respect to the optimal Arrow-security purchase is:

$$\begin{gathered} \max_{D_{t+1}(\varsigma)} \left\{u\left(C_t, N_{ht|t-k}\right) + E_t \beta u\left(C_{t+1}, N_{ht+1|t-k}\right)\right\} \\ \text{subject to} \\ P_t C_t + \int_{\varsigma\in\Omega} V_{t,t+1}(\varsigma) D_{t+1}(\varsigma) d\varsigma = D_t + \int_0^1 W_{ht} N_{iht|t-k} di + T_t, \\ P_{t+1} C_{t+1} + \int_{\varsigma\in\Omega} V_{t,t+2} D_{t+2}(\varsigma) d\varsigma = \int_{\varsigma\in\Omega} \xi_{t,t+1} D_{t+1}(\varsigma) d\varsigma \\ + \int_0^1 W_{ht+1} N_{hit+1|t-k} di + T_{t+1} \end{gathered} \tag{9}$$

When the constraints are inserted into the maximum, the first order condition yields $-u_{ct}\frac{V_{t,t+1}}{P_t} + u_{ct+1}\frac{\xi_{t,t+1}}{P_{t+1}} = 0$. The standard Euler equation emerges by using $Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}}$:

$$Q_t = E_t\left(\frac{u_{ct+1}}{u_{ct}} \frac{P_t}{P_{t+1}}\right) \tag{10}$$

where $Q_t \equiv E_t Q_{t,t+1}$ is defined as the price of the one-period Arrow portfolio. With (2) we get:

$$Q_t = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{11}$$

Finally, a log-linearization yields (lowercase denotes the log of a variable):

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} \left(i_t - \rho - E_t \pi_{t+1} \right) \tag{12}$$

where $i_t \equiv -lnQ_t$, $\rho \equiv -ln\beta$, and CPI inflation is defined as $\pi_t \equiv p_t - p_{t-1}$.

2.1.3 OPTIMAL WAGE SETTING

In contrast to goods, labor services are only traded domestically. Households have monopoly power in the labor market because they specialize in different labor services, while at the same time firms need all kinds of labor to produce goods. Although households set wages, their wage setting decisions are restricted a la Calvo (1983). A constant fraction θ_w of the households is stuck with the wage they had last period, while the remaining $1-\theta_w$ households can reoptimize the price of labor. Thus, a household who resets its wage in period t will choose the optimal wage, denoted $\bar{W}_t$, in order to maximize:[2]

$$\begin{gathered} \max_{\bar{W}_t} E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k u\left(C_{t+k}, N_{ht+k|t}\right) \right\} \\ \text{subject to} \\ N_{ht+k|t} = \left(\frac{\bar{W}_t}{W_{t+k}} \right)^{-\epsilon_w} N_{t+k}, \\ P_{t+k}C_{t+k} + E_{t+k}Q_{t+k,t+k+1}D_{t+k+1|t} = D_{t+k|t} + \bar{W}_t N_{t+k|t} + T_{t+k} \end{gathered} \tag{13}$$

The first constraint is the labor demand from firms (it is explained further below). $N_{ht+k|t}$ is the amount of labor done in time $t+k$ for a household with wage last set in period t. Solving the first order condition for $\bar{W}_t$, the optimal wage schedule emerges:

$$\bar{W}_t = \frac{\epsilon_w}{\epsilon_w - 1} \frac{E_t \sum_{k=0}^{\infty} \left[(\beta\theta_w)^k W_{t+k}^{\epsilon_w} N_{t+k} u_C\left(C_{t+k}, N_{ht+k|t}\right) MRS_{ht+k|t} \right]}{E_t \sum_{k=0}^{\infty} \left[(\beta\theta_w)^k W_{t+k}^{\epsilon_w} u_C\left(C_{t+k}, N_{ht+k|t}\right) \frac{N_{t+k}}{P_{t+k}} \right]} \tag{14}$$

The economy wide time t employment is $N_t \equiv \int_0^1 N_{it} di$. The marginal rate of substitution between consumption and labor in period $t+k$ for a household resetting the wage in period t is denoted $MRS_{ht+k|t} \equiv -\frac{u_N\left(C_{t+k}, N_{ht+k|t}\right)}{u_C\left(C_{t+k}, N_{ht+k|t}\right)}$. In the case with flexible wages ($\theta_w = 0$), the wage setting problem becomes static and (14) collapses to:

$$\frac{W_t}{P_t} = \frac{\epsilon_w}{\epsilon_w - 1} MRS_{t|t} \tag{15}$$

[2]Equation (13) can be interpreted as the expected discounted sum of utilities generated over the uncertain period in which the wage remains unchanged (at the level $\bar{W}_t$ set the current period). Note that the utility generated under any other wage set in the future is irrelevant from the point of view of the optimal setting of the current wage.

Here, symmetry of the model delivers $\bar{W}_t = W_t$ and $MRS_{ht} = MRS_t$ at all times. Thus, $\frac{\epsilon_w}{\epsilon_w - 1}$ is the wedge between the real wage and the marginal rate of substitution that prevails in the absence of wage rigidities. A log-linearization of (14) yields:

$$\bar{w}_t = \mu_w + (1 - \beta\theta_w) E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(mrs_{ht+k|t} + p_{t+k}\right) \tag{16}$$

with $\mu_w \equiv (w - p) - mrs = ln\frac{\epsilon_w}{\epsilon_w - 1}$. The intuition behind the wage setting rule is straight forward. First, $\bar{w}_t$ is increasing in expected future prices because households care about the purchasing power of their nominal wage. Second, $\bar{w}_t$ is increasing in the expected average marginal disutilities of labor in terms of goods over the life of the wage, because households want to adjust their expected average real wage accordingly, given expected future prices. Using (2), one can rewrite (16) to a first order difference equation:

$$\bar{w}_t = \beta\theta_w E_t \bar{w}_{t+1} + (1 - \beta\theta_w) \left(w_t - \frac{\hat{\mu}_{wt}}{1 + \epsilon_w \varphi}\right) \tag{17}$$

The deviation of the economy's average markup from the steady state is denoted $\hat{\mu}_{wt} \equiv \mu_{wt} - \mu_w$.

2.2 Firms

The goods market is characterized by monopoly supply power and sticky prices in a way analogous to the labor market. Firm i's output is given by the following production function:

$$Y_{it} = A_t N_{it}^{1-\alpha} \tag{18}$$

A continuum of differentiated labor services is assumed, all of which are used by each firm. Each household specializes in one type of labor. Thus, total labor N_{it} used by firm i, where N_{hit} is the quantity of type h-labor employed, is defined by:

$$N_{it} \equiv \left(\int_0^1 N_{hit}^{\frac{\epsilon_w - 1}{\epsilon_w}} dh\right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{19}$$

The elasticity of substitution among labor types is denoted ϵ_w. (19) implies that labor is an imperfect substitute as long as $\epsilon_w \ll \infty$.

2.2.1 Optimal labor demand

Firms face a multilevel maximization problem analogous to the consumption problem of households. First, for any given level of labor costs, firms have to find the output maximizing combination of labor. Second, and given this optimal labor vector, firms set prices such that profit is maximized. The solution to the first problem is analogous to the optimal consumption vectors showed earlier. In particular, firm i's demand for labor type h, and the corresponding domestic wage index, are given by:

$$N_{hit} = \left(\frac{W_{ht}}{W_t}\right)^{-\epsilon_w} N_{it}, \quad W_t \equiv \left(\int_0^1 W_{ht}^{1-\epsilon_w} dh\right)^{\frac{1}{1-\epsilon_w}} \tag{20}$$

This holds for all $i, h \in [0, 1]$. In exactly same manner as the aggregation of consumption expenditures, one can derive that firm$i$'s total labor costs are $\int_0^1 W_{ht} N_{hit} dh = W_t N_{it}$.

2.2.2 Optimal price setting

Denote $Y_{it+k|t}$ as the output in period $t+k$ for a firm that last set its price in period t, and $\bar{P}_t$ as the optimal price from the period t point of view. Denote θ_p as the price stickiness parameter in the goods market. Then, the profit maximization problem writes as:

$$\max_{\bar{P}_t} E_t \left\{ \sum_{k=0}^{\infty} \theta_p^k Q_{t,t+k} \left[\bar{P}_t Y_{it+k|t} - \Phi\left(Y_{it+k|t}\right) \right] \right\}$$

subject to

$$Y_{it+k|t} = C_{Hit+k} + \int_0^1 C^j_{Hit+k} dj = \left(\frac{\bar{P}_t}{P_{Ht+k}} \right)^{-\epsilon_p} \left(C_{Ht+k} + \int_0^1 C^j_{Ht+k} dj \right), \tag{21}$$
$$Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}}$$

$\Phi\left(Y_{it+k|t}\right)$ is total nominal costs in period $t+k$ for a firm with price last set in period t. Market clearing in the goods market implies, using (5), the first constraint in (21). The stochastic discount factor $Q_{t,t+k}$ is given by the Euler equation (11). The solution to (21) is a weighted sum of expected future real marginal costs $MC^r_{it+k|t}$:

$$\bar{P}_t = \frac{\epsilon_p}{\epsilon_p - 1} \frac{E_t \sum_{k=0}^{\infty} \left[(\beta\theta_p)^k C_{t+k}^{-\sigma} \left(C_{Ht+k} + \int_0^1 C^j_{Ht+k} dj \right) \frac{1}{P_{t+k}} P_{Ht+k}^{1+\epsilon_p} MC^r_{it+k|t} \right]}{E_t \sum_{k=0}^{\infty} \left[(\beta\theta_p)^k C_{t+k}^{-\sigma} \left(C_{Ht+k} + \int_0^1 C^j_{Ht+k} dj \right) \frac{1}{P_{t+k}} P_{Ht+k}^{\epsilon_p} \right]} \tag{22}$$

In the case with flexible prices ($\theta_p = 0$), (21) collapses to a static problem and (22) becomes $\bar{P}_t = \frac{\epsilon_p}{\epsilon_p - 1} P_{Ht} MC^r_{it|t}$. Then, $\bar{P}_t = P_{Ht}$ and $MC^r_{it|t} \equiv MC^r_t$ because of symmetry, and we get:

$$MC^r_t = \frac{\epsilon_p - 1}{\epsilon_p} \tag{23}$$

Since the real marginal cost is $MC^r_{it} \equiv \frac{1}{P_{Hit}} \frac{\partial \Phi(Y_{it})}{\partial Y_{it}} = \frac{(1-\tau)W_t}{P_{Hit} MPN_{it}}$[3], where τ is a labor wage subsidy yet to be determined, the above gives the net (firm) real wage in the flexible price equilibrium as:

$$\frac{(1-\tau) W_t}{P_{Ht}} = \frac{\epsilon_p - 1}{\epsilon_p} MPN_{it} \tag{24}$$

A log-linearization of (22) yields the following optimal pricing rule:

$$\bar{p}_t = \mu_p + (1 - \beta\theta_p) E_t \sum_{k=0}^{\infty} (\beta\theta_p)^k \left(mc^r_{it+k|t} + p_{Ht+k} \right) \tag{25}$$

Thus, profit maximizing firms will set a price that corresponds to the desired markup, given by $mc^r \equiv \ln(1-\tau) + w - p_H - mpn \equiv -\mu_p$ over a weighted average of their current and expected marginal costs[4], with the weights being proportional to the probability of the price remaining effective at each horizon. (25) can be written as a first order difference equation where $\hat{mc}^r_t$ is the economy's real marginal cost gap:

$$\bar{p}_t = \beta\theta_p E_t \bar{p}_{t+1} + (1 - \beta\theta_p)\left(p_{Ht} + \Theta \hat{mc}^r_t \right) \tag{26}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p} < 1$.

[3]Note that $\Phi(Y_{it}) = (1-\tau) W_t N_{it}$ and (18) imply the nominal marginal cost $\frac{\partial \Phi(Y_{it})}{\partial Y_{it}} = \frac{(1-\tau)W_t}{(1-\alpha)A_t N_{it}^{-\alpha}}$.

[4]Because $MC^r = \frac{\epsilon_p - 1}{\epsilon_p}$, we have that $mc^r = -ln\frac{\epsilon_p}{\epsilon_p - 1} \equiv -\mu_p$.

2.3 THE OPEN ECONOMY

Bilateral terms of trade between the domestic economy and country j is defined as the price of country j's goods in terms of home goods, $S_{jt} \equiv \frac{P_{jt}}{P_{Ht}}$. Thus, the effective terms of trade is given by:

$$S_t = \frac{P_{Ft}}{P_{Ht}} = \left(\int_0^1 S_{jt}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}$$

Combining this with the CPI, one gets the following log-linearized relation between the two:

$$p_t \approx (1-\vartheta) p_{Ht} + \vartheta p_{Ft} = p_{Ht} + \vartheta s_t \tag{27}$$

By the domestic inflation definition $\pi_{Ht} \equiv p_{Ht} - p_{Ht-1}$, CPI inflation and imported inflation follow as:

$$\begin{aligned} \pi_t = p_t - p_{t-1} = \pi_{Ht} + \vartheta \Delta s_t, \\ \pi_{Ft} = p_{Ft} - p_{Ft-1} = \pi_{Ht} + \Delta s_t \end{aligned} \tag{28}$$

Under the assumption that the law of one price holds, the (log) effective nominal exchange rate, $e_t = \int_0^1 e_{jt} dj$, is determined by the terms of trade, the world price level p_t^*, and domestic prices:

$$s_t = p_{Ft} - p_{Ht} = e_t + p_t^* - p_{Ht} \tag{29}$$

The (log) effective real exchange rate, $q_t \equiv \int_0^1 q_{jt} dj$, follows as:

$$q_t = e_t + p_t^* - p_t = s_t + p_{Ht} - p_t = (1-\vartheta) s_t \tag{30}$$

Using the results above, as well as the Euler equation for a generic country j, one arrives at a relationship between domestic consumption and world consumption:

$$c_t = \int_0^1 \left(c_t^j + \frac{1}{\sigma} q_{jt} \right) dj = c_t^* + \frac{1}{\sigma} q_t = c_t^* + \frac{1-\vartheta}{\sigma} s_t \tag{31}$$

The result rests on an assumption of complete asset markets at the international level. Note that the relative home consumption level (to world consumption) is given by the terms of trade. Next we have the uncovered interest rate parity equation, which states that the nominal domestic interest rate is equal to the world nominal rate plus expected rate of depreciation of the home currency. It is found from the optimality conditions with respect to domestic and foreign bonds:

$$i_t = i_t^* + E_t \Delta e_{t+1} \tag{32}$$

Combining this with (29) results in a forward looking relationship between the terms of trade and interest rate differentials:

$$s_t = \left(i_t^* - E_t \pi_{t+1}^* \right) - \left(i_t - E_t \pi_{Ht+1} \right) + E_t s_{t+1} = r_t^* - r_t + E_t s_{t+1} \tag{33}$$

Given stationarity assumptions and a unique solution for s_t in the perfect foresight steady state, (33) shows that terms of trade are determined by current and expected real rate differentials.

2.4 Equilibrium

2.4.1 Equilibrium with sticky wages and prices

Market clearing for good i in the home economy implies the following condition:

$$Y_{it} = C_{Hit} + \int_0^1 C_{Hit}^j dj \tag{34}$$

The domestic demand of good i is denoted C_{Hit} while country j's demand for good i produced in the home economy is denoted C_{Hit}^j. Aggregate domestic output is defined as:

$$Y_t \equiv \left(\int_0^1 Y_{it}^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

Combining with the optimal consumption schedules in home and a generic country j;

$$C_{Hit} = \left(\frac{P_{Hit}}{P_{Ht}} \right)^{-\epsilon_p} C_{Ht} = (1 - \vartheta) \left(\frac{P_{Hit}}{P_{Ht}} \right)^{-\epsilon_p} \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} C_t$$

and

$$C_{Hit}^j = \vartheta \left(\frac{P_{Hit}}{P_{Ht}} \right)^{-\epsilon_p} \left(\frac{P_{Ht}}{\mathcal{E}_{jt} P_{Ft}^j} \right)^{-\gamma} \left(\frac{P_{Ft}^j}{P_t^j} \right)^{-\eta} C_t^j,$$

(where $\mathcal{E}_{jt}$ is the bilateral nominal exchange rate,) equilibrium in the domestic goods market follows as:

$$Y_t = \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} C_t \left[(1 - \vartheta) + \vartheta \int_0^1 \left(S_t^j S_{jt} \right)^{\gamma - \eta} \mathcal{Q}_{jt}^{\eta - \frac{1}{\sigma}} dj \right] \tag{35}$$

The bilateral real exchange rate between the home economy and country j is $\mathcal{Q}_{jt} \equiv \frac{\mathcal{E}_{jt} P_{jt}^j}{P_t}$, i.e. the ratio of the two countries' CPI. A log-linearization around a symmetric steady state (with all prices equal) leads to the following equilibrium condition:

$$y_t = c_t + \frac{\vartheta \delta}{\sigma} s_t \tag{36}$$

where $\delta \equiv \sigma\gamma + (1 - \vartheta)(\sigma\eta - 1)$. A condition analogous to (36) will hold for all countries. Thus, market clearing at the world level implies $y_t^* = c_t^*$. Using this together with (31), (36) becomes:

$$y_t = y_t^* + \frac{1}{\sigma_\vartheta} s_t \tag{37}$$

where $\sigma_\vartheta \equiv \frac{\sigma}{1 + \vartheta(\delta - 1)}$. Combining (36) with the Euler equation, on gets the IS equation:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma_\vartheta} \left(i_t - E_t \pi_{Ht+1} - \rho \right) + \vartheta (\delta - 1) E_t \Delta y_{t+1}^* \tag{38}$$

The last term, $E_t \Delta y_{t+1}^*$, is exogenous to domestic allocations. Note that (38) nests the closed economy case where $\vartheta = 0$. In order to characterize the trade balance, we define net exports nx_t as the difference between domestic production and consumption, relative to steady state output:

$$nx_t \equiv \frac{Y_t - \frac{P_t}{P_{Ht}} C_t}{Y} \tag{39}$$

A first-order approximation around a symmetric steady state with zero net export yields:

$$nx_t \approx y_t - c_t - p_t + p_{Ht} = y_t - c_t - \vartheta s_t = \vartheta \left(\frac{\delta}{\sigma} - 1 \right) s_t \tag{40}$$

The last equality follows from (36). Note for future reference that if $\sigma = \eta = \gamma = 1$, then $nx_t = 0 \, \forall \, t$. Moving on to the labor market, it clears when:

$$N_t = \int_0^1 N_{it} di = \int_0^1 \left(\frac{Y_{it}}{A_t} \right)^{\frac{1}{1-\alpha}} di \tag{41}$$

Equation (41) implies an equivalent relationship between aggregate output and employment as for the single firm (up to a first order):

$$y_t = a_t + (1 - \alpha) \, n_t \tag{42}$$

2.4.2 The natural equilibrium

In this section I summarize some key measures in the equilibrium that would prevail if all households and firms were free to set wages and prices each period. That equilibrium is labeled the natural equilibrium (with superscript n on variables). The linearized real marginal cost in the natural equilibrium follows from (24) as:

$$mc^r = -\mu_p = -\nu + w_t^n - p_{Ht}^n - mpn_t^n \tag{43}$$

The wage subsidy is defined as $\nu \equiv -\ln{(1 - \tau)}$. The natural real marginal cost is equal to the constant $-\mu_p \equiv -ln\frac{\epsilon_p}{\epsilon_p - 1}$ in (23). Equation (43) can be rewritten using (27) and (37):

$$mc^r = -\mu_p = -\nu + \omega_t^n + \left(\vartheta\sigma_\vartheta + \frac{\alpha}{1-\alpha} \right) y_t^n - \vartheta\sigma_\vartheta y_t^* - \frac{1}{1-\alpha} a_t - \ln{(1-\alpha)} \tag{44}$$

Here, the natural real wage is defined as $\omega_t^n \equiv w_t^n - p_t^n$. Notice that this is the real wage from the households' point of view because p_t^n is the (natural) CPI level. The appropriate real wage for the firm is $w_t^n - p_{Ht}^n$. Notice that the economy wide (linearized) version of (15) is $\omega_t^n = \mu_w + mrs_t^n$. Use that observation together with (27), (36), (37) and (42) to derive the natural level of output:

$$y_t^n = \psi_y^n + \psi_{ya}^n a_t + \psi_{y*}^n y_t^* \tag{45}$$

where $\psi_y^n \equiv \frac{(1-\alpha)[\nu - \mu_p - \mu_w + \ln{(1-\alpha)}]}{\varphi + \alpha + \sigma_\vartheta(1-\alpha)}$, $\psi_{ya}^n \equiv \frac{1+\varphi}{\varphi+\alpha+\sigma_\vartheta(1-\alpha)}$, and $\psi_{y*}^n \equiv -\frac{(1-\alpha)\vartheta(\delta-1)\sigma_\vartheta}{\varphi+\alpha+\sigma_\vartheta(1-\alpha)}$. The natural real wage follows by solving (44) for ω_t^n. Substituting for y_t^n we get:

$$\omega_t^n = \psi_w^n + \psi_{wa}^n a_t + \psi_{w*}^n y_t^* \tag{46}$$

where $\psi_w^n \equiv \left[\nu - \mu_p + \ln(1-\alpha) - \left(\vartheta\sigma_\vartheta + \frac{\alpha}{1-\alpha}\right)\psi_y^n\right]$, $\psi_{wa}^n \equiv \left[-\vartheta\sigma_\vartheta\psi_{ya}^n + \frac{1-\alpha\psi_{ya}^n}{1-\alpha}\right]$, and $\psi_{w*}^n \equiv \left[\vartheta\sigma_\vartheta\left(1-\psi_{y*}^n\right) - \frac{\alpha\psi_{y*}^n}{1-\alpha}\right]$. Note for future reference that first differencing yields:

$$\Delta\omega_t^n = \psi_{wa}^n\Delta a_t + \psi_{w*}^n\Delta y_t^* \tag{47}$$

The natural equilibrium block of the model is closed once the exogenous time series a_t and y_t^* are specified. Notice that this block is exogenous to monetary policy behavior.

2.5 Inflation and output

2.5.1 New Keynesian Phillips curves

I shall now derive the relevant forward looking inflation equations for this small open economy. Consider first the New Keynesian wage Phillips curve. Remember that a fraction $1-\theta_w$ of households can reset wages in period t while the remaining θ_w households are stuck with the wage they had last period. Given this structure, the aggregate wage index evolves according to:

$$W_t = \left[\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)\bar{W}_t^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}} \tag{48}$$

Log-linearization around a zero wage inflation steady state yields:

$$w_t \approx \theta_w w_{t-1} + (1-\theta_w)\bar{w}_t \tag{49}$$

Thus, wage inflation can be written as:

$$\pi_{wt} \equiv w_t - w_{t-1} = (1-\theta_w)(\bar{w}_t - w_{t-1}) \tag{50}$$

Combining (17), (49) and (50) one obtains:

$$\pi_{wt} = \beta E_t\pi_{wt+1} - \lambda_w\hat{\mu}_{wt} \tag{51}$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$. Thus, whenever the average wage in the economy is below the level consistent with maintaining the desired markup μ_w, readjusting households will tend to increase their nominal wage, thus generating positive wage inflation. The imperfect adjustment of nominal wages will generally drive a wedge between the real wage and the marginal rate of substitution for each household, and as a result, between the average real wage and the average marginal rate of substitution. This leads to variation in the average wage markup, and given (51), also to variation in wage inflation. The deviation in the economy's average wage markup from its steady state counterpart, where the wage markup writes as $\mu_{wt} \equiv (w_t - p_t) - mrs_t = \omega_t - mrs_t$, is:

$$\hat{\mu}_{wt} \equiv \mu_{wt} - \mu_w = \tilde{\omega}_t - \left[(1-\vartheta)\sigma_\vartheta + \frac{\varphi}{1-\alpha}\right]\tilde{y}_t \tag{52}$$

Here, the real wage gap from its steady state counterpart is defined as $\tilde{\omega}_t \equiv \omega_t - \omega_t^n$ while the corresponding output gap is defined as $\tilde{y}_t \equiv y_t - y_t^n$. Equations (51) and (52) result in the New Keynesian Wage Phillips curve:

$$\pi_{wt} = \beta E_t\pi_{wt+1} + \kappa_w\tilde{y}_t - \lambda_w\tilde{\omega}_t \tag{53}$$

where $\kappa_w \equiv \lambda_w \left[(1-\vartheta)\,\sigma_\vartheta + \frac{\varphi}{1-\alpha}\right]$. In addition, there is an identity linking the real wage gap to the natural real wage inflation $\Delta\omega_t^n$. From the definition $\Delta\tilde{\omega}_t = \Delta\omega_t - \Delta\omega_t^n = \pi_{wt} - \pi_t - \Delta\omega_t^n$:

$$\tilde{\omega}_t \equiv \tilde{\omega}_{t-1} + \pi_{wt} - \pi_t - \Delta\omega_t^n \tag{54}$$

The last term in (54) is exogenous to monetary policy in that it is completely determined by (47).

Next I describe the New Keynesian price Phillips curve in the small open economy. It only characterizes domestic price inflation, CPI inflation is determined by (28). As an analogy to wages, the domestic price level index evolves according to:

$$P_{Ht} = \left[\theta_p P_{Ht-1}^{1-\epsilon_p} + (1-\theta_p)\,\bar{P}_t^{1-\epsilon_p}\right]^{\frac{1}{1-\epsilon_p}} \tag{55}$$

Log-linearization around a zero inflation steady state yields:

$$p_t \approx \theta_p p_{t-1} + (1-\theta_p)\,\bar{p}_t \tag{56}$$

Thus, domestic inflation is an increasing function of $\bar{p}_t$:

$$\pi_{Ht} \equiv p_{Ht} - p_{Ht-1} = (1-\theta_p)\,(\bar{p}_t - p_{Ht-1}) \tag{57}$$

Combining (56) and (57) with (26) one obtains:

$$\pi_{Ht} = \beta E_t \pi_{Ht+1} + \lambda_p \hat{mc}_t^r \tag{58}$$

where $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\Theta$ and Θ is defined as in (26). Notice that the real marginal cost is defined as $mc_t^r = -\nu + w_t - p_{Ht} - mpn_t$. Inserting (27), (36), (37), (42) and (52), the economy's average real marginal cost gap $\hat{mc}_t^r \equiv -\hat{\mu}_{pt}$ is determined:

$$\hat{mc}_t^r = \tilde{\omega}_t + \left(\vartheta\sigma_\vartheta + \frac{\alpha}{1-\alpha}\right)\tilde{y}_t \tag{59}$$

Substituting (59) and (52) into (58), the New Keynesian Phillips curve emerges:

$$\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t \tag{60}$$

where $\kappa_p \equiv \lambda_p \left(\vartheta\sigma_\vartheta + \frac{\alpha}{1-\alpha}\right)$. Finally, one can define a composite inflation measure as a weighted average of price and wage inflation:[5]

$$\pi_{ct} = \left(1 - \frac{\lambda_p}{\lambda_p + \lambda_w}\right)\pi_{Ht} + \frac{\lambda_p}{\lambda_p + \lambda_w}\pi_{wt}$$

Inserting (60) and (53), the following composite New Keynesian Phillips curve emerges:

$$\pi_{ct} = \beta E_t \pi_{ct+1} + \kappa_c \tilde{y}_t \tag{61}$$

where $\kappa_c \equiv \frac{\lambda_p\lambda_w}{\lambda_p+\lambda_w}\left(\sigma_\vartheta + \frac{\varphi+\alpha}{1-\alpha}\right)$. The composite inflation measure will be used for policy analysis.

[5] The weights are motivated by the observation that $\frac{\partial\lambda_p}{\partial\theta_p} < 0$ and $\frac{\partial\lambda_w}{\partial\theta_w} < 0$. Thus, the less rigid prices are compared to wages, the more emphasis is put on the latter in the composite inflation measure.

2.5.2 The dynamic IS equation and the natural real interest rate

In this section I derive a dynamic output gap equation of the small open economy, as well as a process describing the domestic natural real rate. The domestic real interest rate with sticky wages and prices is denoted $r_t \equiv i_t - E_t\pi_{Ht+1}$. Using this, (38) can be written as:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma_\vartheta}(r_t - \rho) + \vartheta(\delta - 1) E_t \Delta y^*_{t+1}$$

In a similar vein, the natural output is given as a function of the natural real interest rate r^n_t:

$$y^n_t = E_t y^n_{t+1} - \frac{1}{\sigma_\vartheta}(r^n_t - \rho) + \vartheta(\delta - 1) E_t \Delta y^*_{t+1} \tag{62}$$

A canonical representation of the dynamic IS equation emerges by subtracting (62) from (38):

$$\tilde{y}_t \equiv y_t - y^n_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma_\vartheta}(i_t - E_t\pi_{Ht+1} - r^n_t) \tag{63}$$

Notably, the world output terms cancel out and we get the same IS equation as in the closed economy case, except that σ is replaced by $\sigma_\vartheta \equiv \frac{\sigma}{1+\vartheta(\delta-1)}$. If the elasticity parameters γ and η, and the risk aversion coefficient σ are sufficiently high, then $\delta \equiv \sigma\gamma + (1-\vartheta)(\sigma\eta - 1)$ is larger than unity and $\sigma_\vartheta > \sigma$. In that case, the output gap is more sensitive to real rate deviations from the natural rate (defined by $r_t - r^n_t$) than if the economy was closed. Note also that (63) nests the closed economy case as $\sigma_\vartheta = \sigma$ when $\vartheta = 0$ (this is also true for the inflation equations). The only thing missing in the non-policy block of the model is a process for the natural real interest rate. One can solve for r^n_t by combining (38), (45) and (63). The result is:

$$r^n_t = \rho + \psi^n_{ra} E_t \Delta a_{t+1} + \psi^n_{r*} E_t \Delta y^*_{t+1} \tag{64}$$

where $\psi^n_{ra} \equiv \sigma_\vartheta \psi^n_{ya}$ and $\psi^n_{r*} \equiv \sigma_\vartheta \left[\vartheta(\delta - 1) + \psi^n_{y*}\right]$. Thus, openness generally makes the natural real interest rate depend on expected world output growth, in addition to domestic productivity. As in the closed economy the real rate converges to the discount rate once technology shocks and world output growth is turned off. For that reason it is useful to label $\hat{r}^n_t \equiv r^n_t - \rho$ as the natural real rate gap (from steady state).

2.5.3 Interest rate rules

To close the model we need to specify a process for the interest rate. I will focus on two families of policy rules. The first is strict rules, which set the interest rate such that one of the variables is determined exactly. I will consider each of the following objectives:

$$\pi_{Ht} = 0, \quad \pi_{wt} = 0, \quad \pi_{ct} = 0, \quad \pi_t = 0, \quad \Delta e_t = 0 \tag{65}$$

The second type of rules is flexible Taylor rules, which have in common that the interest rate responds linearly to some specified variables, most often some inflation measures. I will consider special cases of the following interest rate rule:[6]

$$i_t = \rho + \phi_H \pi_{Ht} + \phi_w \pi_{wt} + \phi_c \pi_{ct} + \phi_{CPI} \pi_t + \phi_e \Delta e_t + v_t \tag{66}$$

[6] Due to space limitations and the fact that the output gap is unobservable, I do not include it in (66). The constant is included to be consistent with the dynamic IS equation, which implies that $i_t = \rho$ in the steady state. The residual v_t is included in order to illustrate the possibility of policy shocks.

During simulations I close down different terms by setting all but one of the response coefficients to zero. Notice that one of the rules is to let the exchange rate be semi-floating. Motivation is found in previous research which suggests that some central banks react in a Taylor-type manner to nominal exchange rate changes (Lubik and Schorfheide, 2007). The four equations (53), (54), (60) and (63) together with the monetary policy constitute the New Keynesian model for a small open economy with sticky wages and prices. Time series movements in the model stem from exogenous innovations in the domestic technology level a_t and the world output level y_t^* (and possible policy shocks). Those innovations are transmitted into the economy through changes in the natural real rate, the natural output level and the natural real wage.

3 POLICY ANALYSIS

3.1 THE ROLE OF FISCAL POLICY

There are two kinds of distortions in this small open economy model. One is market power, both among households and firms, the other is sticky wages and prices. Market power, which stems from the construction of imperfect elastic demand, gives individual households and firms the opportunity to set a markup on competitive wages and prices. In the following I describe how fiscal policy can be used to remove that distortion. I follow GM and make the following parameter assumptions:

$$\sigma = \eta = \gamma = 1$$

These assumptions simplify some of the equations. In particular, the CPI takes the Cobb-Douglas form $P_t = P_{Ht}^{\vartheta} P_{Ft}^{1-\vartheta}$. Goods market clearing and domestic consumption are now given by:

$$Y_t = C_t S_t^{\vartheta} \tag{67}$$

$$C_t = Y_t^{1-\vartheta} Y_t^{*\vartheta} \tag{68}$$

What is the optimal allocation from the social planner's view? By symmetry of the model, he finds it optimal to let $N_{it} = N_t$ and $Y_{it} = Y_t$. Thus, domestic consumption writes as:

$$C_t = \left(A_t N_t^{1-\alpha}\right)^{1-\vartheta} Y_t^{*\vartheta} \tag{69}$$

The (period-by-period) problem of the social planner becomes a problem in N_t only:

$$\max_{N_t} \left\{ u \left[\left(A_t N_t^{1-\alpha}\right)^{1-\vartheta} Y_t^{*\vartheta}, N_t \right] \right\} \tag{70}$$

The relevant optimality condition is:

$$MRS_t \equiv -\frac{u_{Nt}}{u_{Ct}} = (1-\alpha)(1-\vartheta)\frac{C_t}{N_t} = (1-\vartheta) MPN_t \tag{71}$$

The last equality follows from the parameter assumption, which gives zero net export at all times, implying that domestic consumption equals domestic production.[7] With $\sigma = 1$,

[7] The specified production function implies that $MPN_t = (1-\alpha) A_t N_t^{-\alpha} = (1-\alpha)\frac{Y_t}{N_t} = (1-\alpha)\frac{C_t}{N_t}$.

which implies that $u(C_t, N_t) = \ln C_t - \frac{N_t^{1+\varphi}}{1+\varphi}$, (71) can be solved for N_t:

$$N_t = [(1-\alpha)(1-\vartheta)]^{\frac{1}{1+\varphi}} \tag{72}$$

Thus, the socially optimal employment level is constant. Notice that (72) captures as a limiting case the CRS assumption made by GM (they find the optimal employment as) $(1-\vartheta)^{\frac{1}{1+\varphi}}$. In contrast to (71) however, the natural equilibrium delivers wages and prices according to (15) and (24). It follows from balanced trade and (67) that (24) can be written as $\frac{(1-\tau)W_t}{P_{Ht}} = \frac{\epsilon_p - 1}{\epsilon_p} MPN_t$ ($S_t = 1\ \forall\ t$ in this case). Together, (15) and (24) give the following natural equilibrium condition:

$$-\frac{u_{Nt}}{u_{Ct}} = \frac{1}{1-\tau}\frac{\epsilon_p - 1}{\epsilon_p}\frac{\epsilon_w - 1}{\epsilon_w} MPN_t \tag{73}$$

By comparing (73) with (71), the method of undetermined coefficients delivers the following optimal wage subsidy:

$$\tau = \frac{1}{1-\vartheta}\left(\frac{\epsilon_p + \epsilon_w - 1}{\epsilon_p \epsilon_w} - \vartheta\right) \tag{74}$$

Thus, once nominal rigidities are taken care of, (74) assures that the natural equilibrium is also the efficient one. Note that (74) nests the closed economy case where $\vartheta = 0$. In that case, the optimal subsidy collapses to $\tau = \frac{\epsilon_p + \epsilon_w - 1}{\epsilon_p \epsilon_w} > 0$ as in Galí (2008), ch. 6. However, because $0 \leq \vartheta \leq 1$, a sufficiently open economy in combination with a sufficiently high substitability between consumption goods and between labor types implies that $\tau < 0$. Under the calibration used here, this is exactly what happens. Therefore, in the open economy a payroll tax can actually be the optimal fiscal policy instead of a wage subsidy.

3.2 Wage rigidity and the monetary policy tradeoff

It is well known that the most basic version of the New Keynesian model, the one with a closed economy and flexible wages, does not deliver a meaningful tradeoff between inflation and output gap stabilization for monetary authorities.[8] By responding sufficiently aggressive to price changes in that model, the output gap from the efficient (or in our case natural) level is closed down. This result implies strict (domestic) inflation targeting as the optimal monetary policy. However, real life central banks do indeed pay explicit attention to output. One way to justify this behavior is to add real rigidities to the model, in particular cost push shocks. Another approach, which is the one I follow here, is to impose Calvo setting of prices in the labor market in addition to the goods market.[9] This generates a tradeoff for monetary authorities in the following manner: In order for the constraints on price and wage setting not to be binding, all firms and workers should view their current prices and wages as the desired ones. This makes adjustment unnecessary and leads to constant aggregate price and wage levels, i.e. zero inflation in both markets. Note, however, that such an outcome implies a constant real wage, which will generally

[8] The discussion in this subsection follows closely that of Galí (2008), chapters 5 and 6.

[9] In an open economy setting one can also induce a tradeoff for policy by assuming local currency pricing (LCP) along the lines of Monacelli (2005).

be inconsistent with the flexible price and wage level allocation. Only when the natural real wage is constant, i.e. when $\Delta\omega_t^n = 0 \;\forall\, t$, and at the same time the central bank adjusts the nominal rate one-for-one with changes in the natural real rate, i.e. $v_t = \hat{r}_t^n \;\forall\, t$, $\tilde{y}_t = \pi_{Ht} = \pi_{wt} = 0$ is an obtainable outcome. According to (47) however, constant real wages require complete absence of technology shocks and foreign shocks. This restriction can hardly be justified. As a result, the model delivers a tradeoff for the central bank. That argument holds regardless of the policy rule.

3.3 Monetary policy design

Because monetary authorities cannot implement an interest rate rule which simultaneously stabilizes inflation measures and the output gap, it is important to characterize how monetary policy should be conducted instead. In Appendix A I show that under the assumptions set up in this paper, in particular that of $\sigma = \eta = \gamma = 1$, a second order approximation to the utility losses experienced by the representative household (as a consequence of deviations from the efficient allocation) yields the following welfare loss function:

$$\mathbb{W} = -\frac{1-\vartheta}{2}\sum_{t=0}^{\infty}\beta^t\left[\frac{1+\varphi}{1-\alpha}\tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p}\pi_{Ht}^2 + \frac{\epsilon_w(1-\alpha)}{\lambda_w}\pi_{wt}^2\right] \tag{75}$$

with λ_w and λ_p being defined as in (51) and (58), respectively. Welfare losses are expressed in terms of the equivalent permanent consumption decline, measured as a fraction of the steady state consumption:

$$\mathbb{W} \equiv \sum_{t=0}^{\infty}\beta^t\left(\int_0^1 \frac{u_{ht}-u}{u_C C}dh\right) - t.i.p. - o\left(\|\cdot\|^3\right)$$

where $u_{ht} \equiv u\left(C_{ht|t-k}, N_{ht|t-k}\right)$ is the period utility of household h, and $t.i.p.$ stands for terms independent of policy. Evidently a volatile CPI inflation does not represent any direct welfare losses. In that respect the sticky wage model does not alter the results in GM.[10] Taking unconditional expectations of (75) and letting $\beta \to 1$, the expected period welfare losses can be written in terms of the variances of inflation measures and the output gap:

$$\mathbb{L} = -\frac{1-\vartheta}{2}\left[\frac{1+\varphi}{1-\alpha}var(\tilde{y}_t) + \frac{\epsilon_p}{\lambda_p}var(\pi_{Ht}) + \frac{\epsilon_w(1-\alpha)}{\lambda_w}var(\pi_{wt})\right] \tag{76}$$

I will now perform a normative monetary policy analysis (under the assumption of full commitment). For a number of different monetary policy regimes I simulate innovations to technology and world output, and calculate the (period) welfare losses as implied by equation (76) and the simulated second moments. This exercise may give some insight into how monetary policy should be conducted in our small open economy. The same setup is used as in GM. In particular, shocks in the model are stationary AR(1) processes:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{77}$$
$$y_t^* = \rho_* y_{t-1}^* + \varepsilon_t^* \tag{78}$$

[10] As the CPI inflation is equal to domestic inflation in a closed economy, it would of course show up in the loss function if $\vartheta = 0$. More generally, because CPI inflation is given by $\pi_t = \pi_{Ht} + \vartheta\Delta s_t$, stabilizing the CPI given a fixed terms of trade level is equivalent to stabilizing domestic inflation.

Parameter calibrations are summarized in Appendix B. I follow GM apart from a few exceptions. The exceptions are as follows: First, it is assumed in the GM model that production technology is constant returns to scale. In contrast, I model diminishing returns to scale and set $\alpha = 1/3$. Second, the labor wage subsidy is set to -15.7%. This makes the natural equilibrium socially efficient given the analysis that lead to (74). Notice that the subsidy is in fact a payroll tax.

Table 1: Welfare losses under strict target rules

	(1) Optimal policy		(2) Domestic inflation		(3) Wage inflation		(4) Composite inflation		(5) CPI inflation		(6) Exchange rate peg	
	St.d.	%	St.d.	%	St.d.	%	St.d.	%	St.d.	%	St.d.	%
$\tilde{y}_t$	0.0085	0.10	1.5768	74.28	0.1555	24.13	0	0	1.2069	59.38	1.4089	51.21
π_{Ht}	0.1449	88.64	0	0	0.1616	75.87	0.1449	87.69	0.2422	06.96	0.3330	8.33
π_{wt}	0.0290	11.26	0.3055	25.72	0	0	0.0305	12.31	0.2992	33.66	0.4124	40.46
π_t	1.4721	-	3.3479	-	1.3345	-	1.4383	-	0	-	0.1998	-
Δe_t	0.9327	-	2.0924	-	0.8438	-	0.9121	-	0.0908	-	0	-
Welfare loss	0.0012		0.0602		0.0018		0.0013		0.0442		0.0698	

3.3.1 Strict rules

Simulation results when monetary authorities follow strict rules as in (65) are summarized in Table 1. There are two sub-columns for each rule. The first reports standard deviations (in percent) of different key variables as implied by the simulated model. The percentage share of total losses as implied by (76) is reported in the second sub-column. The optimal policy (column 1) serves as a benchmark comparison. That policy sets the interest rate that maximizes (75) subject to (53), (54) and (60). According to the numerical results, the optimal policy yields a rather trivial welfare decline with most losses coming from domestic inflation. The output gap has nearly zero volatility. This is in stark contrast to a strict domestic inflation target (column 2). That policy yields welfare losses 50 times as high as the optimal policy, with three quarters of the losses coming from the output gap. A strict wage inflation targeting (column 3) performs much better, however. This is not surprising given the calibration, which implies that the weight on wage inflation in the welfare loss function is more than 3 times as high as the weight on domestic inflation.[11] Even lower welfare losses are generated when the central bank completely closes the composite inflation measure (column 4) given in (61). Such a policy does, for all practical purposes, as well as the optimal policy. Notice how this policy, in contrast to all the others, nearly duplicates the optimal policy by a complete closure of the output gap. Some intuition can be drawn from the basic New Keynesian model without cost-push shocks and wage rigidities (see Galí (2008) ch. 3). If the central bank sets an interest rate such

[11]The weights are approximately 885 on wage inflation and 279 on domestic price inflation.

that $\pi_{Ht} = 0$ at all times in that model, then $\tilde{y}_t = 0$ at all times. Here one achieves an analogous output gap closure by always letting the interest rate adjust such that $\pi_{ct} = 0$. Since the optimal policy produces nearly zero output gaps, strict composite inflation targeting becomes a very good approximation, at least in that dimension. Moving on to strict CPI inflation targeting (column 5), it is seen that this rule performs better than a strict domestic inflation target. This finding contrasts one of the central results in GM and vindicates the observation that central banks generally pay attention to the CPI. It should also be noticed that an exchange rate peg (column 6) produces welfare losses quite similar to strict domestic inflation targeting.

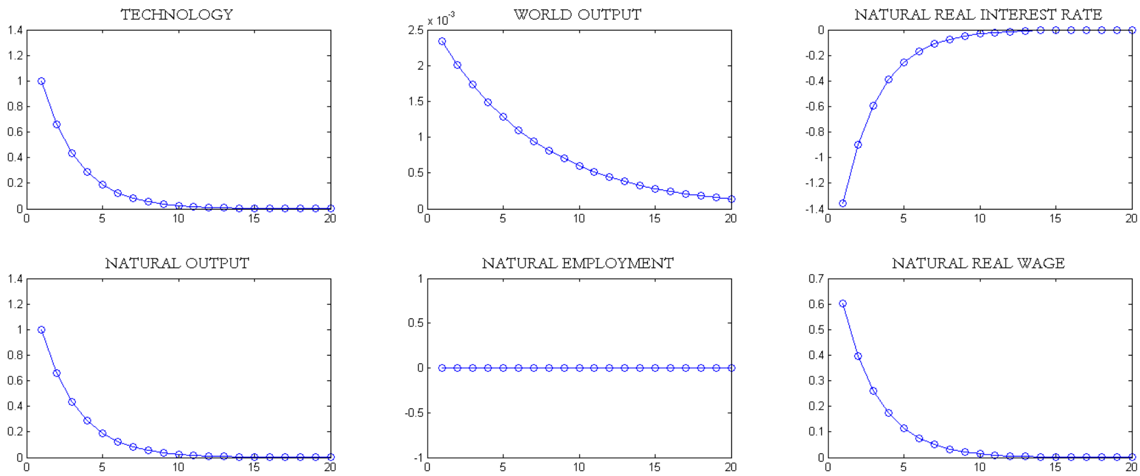
Table 2: Welfare losses under flexible target rules (Taylor rules)

	(7) Optimal Taylor		(8) Domestic inflation		(9) Wage inflation		(10) Composite inflation		(11) CPI inflation		(12) Exchange rate	
ϕ_H	7.63		1.5		-		-		-		-	
ϕ_w	6.82		-		1.5		-		-		-	
ϕ_c	-		-		-		1.5		-		-	
ϕ_{CPI}	3.97		-		-		-		1.5		-	
ϕ_e	-1.53		-		-		-		-		1.5	
	St.d.	%	St.d.	%	St.d.	%	St.d.	%	St.d.	%	St.d.	%
$\tilde{y}_t$	0.0851	6.37	0.7934	49.28	0.8140	60.63	0.4915	51.29	0.9682	18.51	1.2234	9.66
π_{Ht}	0.1610	66.31	0.2883	18.96	0.2547	17.29	0.2174	29.22	0.5583	17.93	1.0287	19.89
π_{wt}	0.0581	27.33	0.2097	31.76	0.1618	22.08	0.0998	19.50	0.5907	63.56	1.0879	70.45
π_t	1.4086	-	1.2849	-	1.2863	-	1.2361	-	1.1763	-	1.1299	-
Δe_t	0.8938	-	0.7729	-	0.7778	-	0.7657	-	0.6166	-	0.3554	-
Welfare loss	0.0020		0.0230		0.0197		0.0085		0.0911		0.2789	

3.3.2 FLEXIBLE RULES

Both the optimal policy and the strict inflation rules can be problematic to implement for several reasons. First of all the optimal policy must satisfy all the first order conditions and the constraints from maximizing (75). That in turn requires complete knowledge about all variables, including unobservables such as Lagrangian multipliers from the constraints. Even if one ignores the optimal policy, but instead settles with a strict rule, implementation still requires knowledge about the natural equilibrium, which really is unobservable. These issues have led researchers to look for flexible monetary rules, most often some kind of Taylor rules where the interest rate responds systematically to observable inflation measures. Table 2 reports how good flexible rules are as approximations to the optimal policy. As a characterization of the best Taylor rule possible (column 7), I first do a numerical search for the optimal coefficients in (66).[12] The resulting losses are almost as small as under strict wage or composite inflation targeting, but the response coefficients are substantially higher than what is typically observed in the empirical literature. In particular, such estimates lie in the neighborhood around 1.5, which is also used as the benchmark calibration in Galí (2008). Thus, the remaining flexible rules are studied using that number. Interestingly, the flexible domestic inflation target (column 8) seems to outperform a complete closure of domestic inflation, and the relative gain of (flexible) wage targeting (column 9) is much smaller than before. However, the composite target (column 10) is still superior. Finally, it is seen that a CPI target (column 11) and an exchange rate target (column 12) are substantially worse when the monetary authorities follow flexible rules.[13] How can the CPI target be so bad when GM conclude that the difference between a CPI target and a domestic target is small and negligible? The answer lies in the stickiness of wages. Letting θ_w approach zero I find very similar welfare losses as GM.[14]

Figure 1: Effects of a technology shock to the natural equilibrium



3.4 Dynamic responses to a technology shock

Given the results in the previous section, it is informative to compare impulse responses from the optimal policy with other rules. Thus, I will simulate a unit innovation to technology. Dynamics in the natural equilibrium are shown first as a basis for comparison in Figure 1. It is seen that the technology shock leads to a rise in natural output. In fact, the two move one-for-one with each other. The reason is that trade is balanced at all times

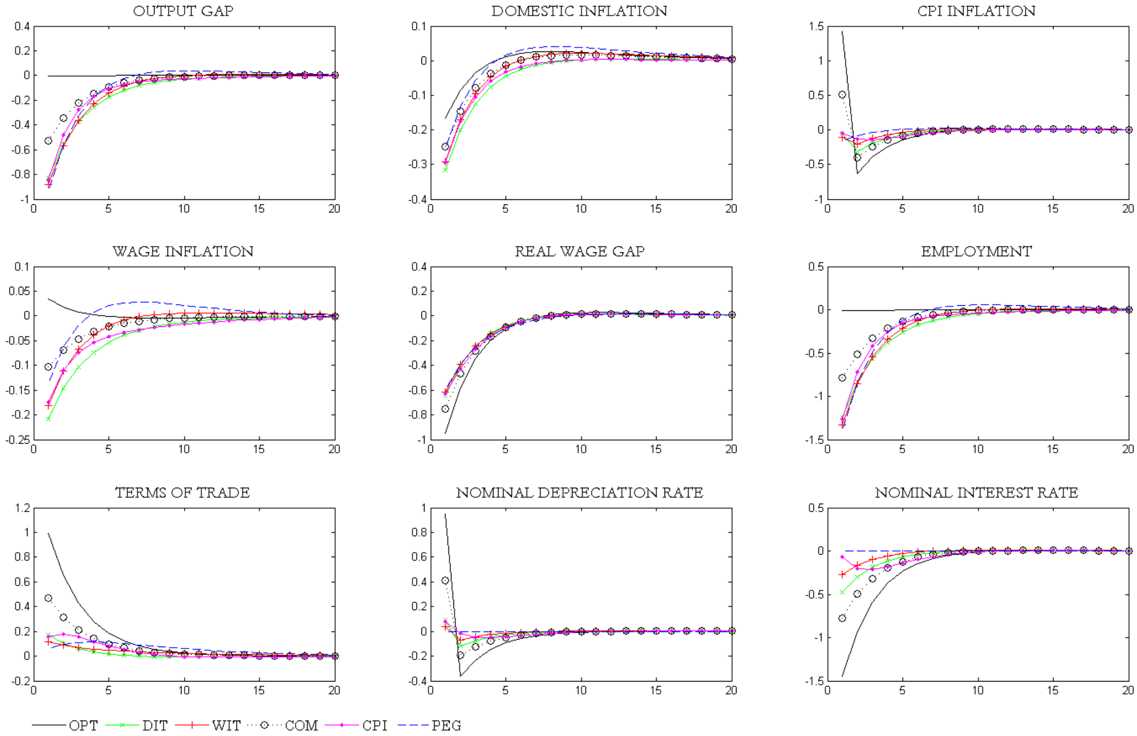
[12] I search for the point in the (ϕ_H, ϕ_w, ϕ_{CPI}, ϕ_e)-space that maximizes (75) subject to (53), (54), (60) and the extended Taylor rule (66). Notice that ϕ_c is not a part of this problem (it is capsulated by ϕ_H and ϕ_w anyway, and therefore set to zero). Response coefficients are initialized at $\phi_H = \phi_w = \phi_{CPI} = \phi_e = 1.5$.

[13] A positive coefficient on the depreciation rate is motivated in the following manner: Suppose the domestic currency is subject to a positive (currency) demand shock, which leads to a currency appreciation (i.e. e_t falls). Then the central bank can reduce the nominal interest rate, and thereby dampen the currency demand.

[14] I also checked whether the results are robust to setting $\alpha = 0$ as in GM. However, diminishing returns do not seem to play an important role (results not reported).

under the specified calibration. Higher output induces a raise in real wages. However, the marginal rate of substitution changes in favor of more leisure (due to reduced marginal utility of consumption). The two effects completely offset each other given the fiscal policy in place, and employment remains unchanged. These natural equilibrium dynamics play out regardless of the monetary policy.

Figure 2: Effects of a technology shock under different monetary policy rules



Impulse responses under wage and price rigidity are shown in Figure 2 for different interest rate rules. In addition to the optimal policy, impulse responses are shown for the flexible Taylor rules, where all but one of the coefficients in (66) are set to zero. The coefficient of interest is kept at 1.5. Instead of the flexible exchange rate target, I simulate an exchange rate peg (PEG) in order to facilitate comparison with GM. The top left panel in Figure 2 illustrates how the optimal monetary policy OPT is able to close the output gap almost completely. This is in stark contrast to the alternative rules. What is the mechanism? Indeed, under OPT there is a small wage increase, but the CPI level rises by far more, implying a negative real wage gap on impact that is not seen with any other rule. Although the CPI starts to revert from the second period and onwards, the real wage gap stays negative for some time due to the large initial rise in consumer prices. To limit the income effect of low real wages, households need to increase the labor supply. Thus, on the one hand leisure has become relatively cheaper. On the other hand, households face an income effect inducing them to work more. The two effects almost neutralize each other under the optimal policy, leading to nearly constant employment. In contrast, the candidate rules are all associated with large employment reductions, and thus with significant negative output gaps. Of these rules, composite inflation targeting COM replicates the optimal rule better than alternatives in all the relevant dimensions. In particular it leads to smaller effects on domestic inflation and wage inflation than domestic inflation targeting DIT and wage inflation targeting WIT, but also to less negative output gaps. This is in line with the results in the tables. The CPI Taylor rule and the PEG policy, which both produce relatively large welfare losses, instead stabilize exchange rates and the terms of trade. However, we have already learned that such moments are irrelevant in the setting considered here.

4 Conclusion

Previous work on small open economy (New Keynesian) models has suggested domestic inflation as an optimal target for monetary authorities. Furthermore, the literature has shown that the standard small open economy model without cost push shocks does not deliver a tradeoff between stabilizing inflation and the output gap. Still, real life central banks do indeed pay attention to the output gap, and they typically target CPI inflation rather than domestic inflation. In this paper I point to wage rigidities as a way to modify previous results. In particular I incorporate the sticky wage setup of Erceg et al. (2000) in an otherwise standard small open economy model, and show that sticky wages deliver a tradeoff between stabilizing domestic prices, wages and the output gap. I derive a second-order approximation to the utility losses experienced by a representative household and analyze a number of different monetary rules. The results suggest that wage inflation targeting may actually be better than targeting domestic price inflation. Thus, if one believes in sticky wages, a feature supported by empirical literature[15], it may be of special importance for monetary authorities to take wage inflation into account when setting interest rates. However, a composite inflation target where weights are put on inflation measures according to their respective rigidities (and some curvature parameters) significantly outperforms other rules. Also, under certain conditions (in particular that of a central bank following strict rules) it may be better to target CPI than domestic inflation.

[15] Barattieri, Basu, and Gottschalk (2010) find support for a high degree of wage stickiness. After correcting for measurement error, the authors estimate a quarterly probability that an individual will experience nominal wage changes between 5 and 18 percent.

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APPENDIX

A AN APPROXIMATION TO WELFARE LOSSES

In the following I will derive a welfare loss function for the special case with log utility and unit elasticity of substitution between goods of different origin (i.e. for $\sigma = \eta = \gamma = 1$). In order to lighten the notation, denote the period t utility as $u_t \equiv u(C_t, N_t)$ and the steady state utility as $u \equiv u(C, N)$. I will frequently use the following second-order approximation to relative deviations in a variable X_t from its steady state counterpart X:

$$\frac{X_t - X}{X} \approx \frac{1}{X} e^x (x_t - x) + \frac{1}{2}\frac{1}{X} e^x (x_t - x)^2 = \hat{x}_t + \frac{1}{2}\hat{x}_t^2$$

A second-order Taylor expansion of the utility function leads to:

$$\begin{aligned} u_{ht} - u &\approx u_C C \frac{C_{ht} - C}{C} + u_N N \frac{N_{ht} - N}{N} \\ &\quad + \frac{1}{2} u_{CC} C^2 \left(\frac{C_{ht} - C}{C}\right)^2 + \frac{1}{2} u_{NN} N^2 \left(\frac{N_{ht} - N}{N}\right)^2 \\ &\approx u_C C \left(\hat{c}_{ht} + \frac{1 + \frac{u_{CC}}{u_c} C}{2} \hat{c}_{ht}^2\right) + u_N N \left(\hat{n}_{ht} + \frac{1 + \frac{u_{NN}}{u_N} N}{2} \hat{n}_{ht}^2\right) \\ \Rightarrow \int_0^1 \frac{u_{ht} - u}{u_C C} dh &\approx \hat{c}_t + \frac{1 + \frac{u_{CC}}{u_c} C}{2} \hat{c}_t^2 \\ &\quad + \frac{u_N}{u_C} \frac{N}{C} \left(\int_0^1 \hat{n}_{ht} dh + \frac{1 + \frac{u_{NN}}{u_N} N}{2} \int_0^1 \hat{n}_{ht}^2 dh\right) \end{aligned}$$

A few results are needed. First, notice that in the special case considered here, (40) can be rewritten to $s_t = y_t - y_t^*$. Thus, (31) becomes $c_t = c_t^* + (1 - \vartheta) s_t = (1 - \vartheta) y_t + \vartheta y_t^*$. Insert that and use that $\frac{u_{CC}}{u_c} C = -1$ in the log consumption case. The result is:

$$\int_0^1 \frac{u_{ht} - u}{u_C C} dh \approx (1 - \vartheta) \hat{y}_t + \frac{u_N}{u_C} \frac{N}{C} \left(\int_0^1 \hat{n}_{ht} dh + \frac{1 + \varphi}{2} \int_0^1 \hat{n}_{ht}^2 dh\right) + t.i.p. \quad \text{(A.1)}$$

$t.i.p.$ stands for terms independent of policy. Next, define aggregate employment $N_t \equiv \int_0^1 N_{ht} dh$. Then, in terms of log deviations from a steady state and up to a second-order approximation we have $\frac{N_{ht} - N}{N} \approx \hat{n}_{ht} + \frac{1}{2}\hat{n}_{ht}^2$ or $\int_0^1 \hat{n}_{ht} dh \approx \hat{n}_t + \frac{1}{2}\hat{n}_t^2 - \frac{1}{2}\int_0^1 \hat{n}_{ht}^2 dh$. Insert this into (A.1):

$$\int_0^1 \frac{u_{ht} - u}{u_C C} dh \approx (1 - \vartheta) \hat{y}_t + \frac{u_N}{u_C} \frac{N}{C} \left(\hat{n}_t + \frac{1}{2}\hat{n}_t^2 + \frac{\varphi}{2} \int_0^1 \hat{n}_{ht}^2 dh\right) + t.i.p. \quad \text{(A.2)}$$

Next, more results are derived. First, a first-order approximation of the demand for labor from household h is $\hat{n}_{ht} = \frac{N_{ht} - N}{N} \approx -\epsilon_w \hat{w}_{ht} + \hat{n}_t$. Second, and using the result above:

$$\int_0^1 \hat{n}_{ht}^2 dh = \int_0^1 (-\epsilon_w \hat{w}_{ht} + \hat{n}_t)^2 dh = \epsilon_w^2 \int_0^1 \hat{w}_{ht}^2 dh - 2\epsilon_w \hat{n}_t \int_0^1 \hat{w}_{ht} dh + \hat{n}_t^2 \quad \text{(A.3)}$$

Third, from the wage index we have that $1 = \int_0^1 e^{(1-\epsilon_w)(w_{ht} - w_t)} dh$. A second-order approximation with respect to w_{ht} yields $\int_0^1 \hat{w}_{ht} dh \approx -\frac{1-\epsilon_w}{2} \int_0^1 \hat{w}_{ht}^2 dh$. Taking expectations on both sides, where E_i denotes the expectations operator with respect to labor for firm i, we get:

$$E_i \hat{w}_{ht} = \int_0^1 \hat{w}_{ht} di \approx -\frac{1-\epsilon_w}{2} E_i \hat{w}_{ht}^2 = -\frac{1-\epsilon_w}{2} var_i w_{ht} \tag{A.4}$$

Thus, to a first order, (A.3) can be written:

$$\int_0^1 \hat{n}_{ht}^2 dh = \hat{n}_t^2 + \epsilon_w^2 \int_0^1 \hat{w}_{ht}^2 dh = \hat{n}_t^2 + \epsilon_w^2 var_h w_{ht}$$

Insert that result into (A.2) and get to:

$$\int_0^1 \frac{u_{ht} - u}{u_C C} dh \approx (1-\vartheta)\, \hat{y}_t + \frac{u_N}{u_C} \frac{N}{C} \left(\hat{n}_t + \frac{1+\varphi}{2} \hat{n}_t^2 + \frac{\epsilon_w^2 \varphi}{2} var_h w_{ht} \right) + t.i.p. \tag{A.5}$$

The next step is to derive a relationship between aggregate employment and output. Using (20), (18), and (5) in that order, and the fact that domestic output is equal to domestic consumption under the current calibration, one gets to:

$$N_t = \Delta_{wt} \Delta_{pt} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}}$$

where $\Delta_{wt} \equiv \int_0^1 \left(\frac{W_{ht}}{W_t} \right)^{-\epsilon_w} dh$ and $\Delta_{pt} \equiv \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{-\frac{\epsilon_p}{1-\alpha}} di$. Take the log, and then a second-order approximation to the relationship between (log) aggregate output and aggregate employment. The result is:

$$(1-\alpha)\, \hat{n}_t \approx \hat{y}_t - a_t + d_{wt} + d_{pt} \tag{A.6}$$

where $d_{wt} \equiv (1-\alpha) \ln \left[\int_0^1 \left(\frac{W_{ht}}{W_t} \right)^{-\epsilon_w} dh \right]$ and $d_{pt} \equiv (1-\alpha) \ln \left[\int_0^1 \left(\frac{P_{Hit}}{P_{Ht}} \right)^{-\frac{\epsilon_p}{1-\alpha}} di \right]$. The next step is to get alternative expressions for d_{wt} and d_{pt}. A second-order approximation of what is inside the integral in d_{pt} leads to:

$$\left(\frac{P_{Hit}}{P_{Ht}} \right)^{-\frac{\epsilon_p}{1-\alpha}} \approx 1 - \frac{\epsilon_p}{1-\alpha} \hat{p}_{it} + \frac{1}{2} \left(\frac{\epsilon_p}{1-\alpha} \right)^2 \hat{p}_{it}^2 \tag{A.7}$$

Using (A.7), a second-order approximation of d_{pt} delivers:

$$d_{pt} \approx \frac{\epsilon_p}{2\Theta} var_i p_{Hit} \tag{A.8}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$. Here I have used a second-order approximation of $\left(\frac{P_{Hit}}{P_{Ht}} \right)^{1-\epsilon_p}$ which yields $1 + (1-\epsilon_p)\, \hat{p}_{it} + \frac{1}{2}(1-\epsilon_p)^2 \hat{p}_{it}^2$. This implies that:

$$E_i \left(\frac{P_{Hit}}{P_{Ht}} \right)^{1-\epsilon_p} = 1 = E_i \left[1 + (1-\epsilon_p)\, \hat{p}_{it} + \frac{1}{2}(1-\epsilon_p)^2 \hat{p}_{it}^2 \right]$$

$$\Rightarrow E_i \hat{p}_{it} = -\frac{1-\epsilon_p}{2} var_i \hat{p}_{it}^2$$

Next I do a similar derivation for d_{wt}. A second-order approximation of $\left(\frac{W_{ht}}{W_t}\right)^{-\epsilon_w}$ in d_{wt}:

$$\left(\frac{W_{ht}}{W_t}\right)^{-\epsilon_w} = 1 - \epsilon_w \hat{w}_{ht} + \frac{1}{2}\epsilon_w^2 \hat{w}_{ht}^2$$

Insert this into d_{wt} and use (A.4). The result is:

$$d_{wt} \approx (1-\alpha)\frac{\epsilon_w}{2} var_h w_{ht} \tag{A.9}$$

Now we are ready to update (A.6):

$$\hat{n}_t \approx \frac{1}{1-\alpha}(\hat{y}_t - a_t) + \frac{\epsilon_w}{2} var_h w_{ht} + \frac{\epsilon_p}{(1-\alpha)\,2\Theta} var_i p_{Ht} \tag{A.10}$$

Substituting this into (A.5) gives:

$$\int_0^1 \frac{u_{ht}-u}{u_C C} dh \approx -\frac{1-\vartheta}{2}\left[\Upsilon var_h w_{ht} + \frac{\epsilon_p}{\Theta} var_i p_{Ht} + \frac{1+\varphi}{1-\alpha}(\hat{y}_t - a_t)^2\right] + t.i.p.$$

where $\Upsilon \equiv \epsilon_w (1-\alpha)(1+\epsilon_w \varphi)$. I have assumed that the optimal fiscal policy is in place, so that $-\frac{u_N}{u_C} = (1-\alpha)(1-\vartheta)\frac{C}{N}$. To proceed, note that the parameter assumptions reduce (45) to:

$$y_t^n = \psi_y^n + \frac{1+\varphi}{\varphi + \alpha + \sigma_\vartheta (1-\alpha)} a_t - \frac{(1-\alpha)\vartheta(\delta-1)\sigma_\vartheta}{\varphi+\alpha+\sigma_\vartheta(1-\alpha)} y_t^* = \psi_y^n + a_t$$

Thus, $\hat{y}_t^n \equiv y_t^n - y^n = a_t$. Insert this into the welfare loss approximation and rewrite:

$$\int_0^1 \frac{u_{ht}-u}{u_C C} dh \approx -\frac{1-\vartheta}{2}\left(\Upsilon var_h w_{ht} + \frac{\epsilon_p}{\Theta} var_i p_{Ht} + \frac{1+\varphi}{1-\alpha}\tilde{y}_t^2\right) + t.i.p. \tag{A.11}$$

Here, I have used that $\hat{y}_t - \hat{y}_t^n = (y_t - y) - (y_t^n - y) = \tilde{y}_t$. The final step consists in rewriting the terms involving price and wage dispersion in (A.11) as functions of inflation rates. Woodford (2003) has shown that:[16]

$$\sum_{t=0}^{\infty} \beta^t var_i p_{Ht} \approx \frac{\theta_p}{(1-\theta_p)(1-\beta\theta_p)} \sum_{t=0}^{\infty} \beta^t \pi_{Ht}^2$$
$$\sum_{t=0}^{\infty} \beta^t var_h w_{ht} \approx \frac{\theta_w}{(1-\theta_w)(1-\beta\theta_w)} \sum_{t=0}^{\infty} \beta^t \pi_{wt}^2$$

When we take the discounted sum of (A.11) over all periods and make use of the two equations above, the result is:

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t \frac{u_t - u}{u_C C} &= \sum_{t=0}^{\infty} \beta^t \left(\int_0^1 \frac{u_{ht}-u}{u_C C} dh\right) \\ &\approx -\frac{1-\vartheta}{2}\sum_{t=0}^{\infty} \beta^t \left[\frac{\epsilon_w (1-\alpha)}{\lambda_w}\pi_{wt}^2 + \frac{\epsilon_p}{\lambda_p}\pi_{Ht}^2 + \frac{1+\varphi}{1-\alpha}\tilde{y}_t^2\right] + t.i.p.\end{aligned}$$

[16] See the proof to proposition 6.3 as well as the results in Woodford (2003, p. 400). A detailed proof is available upon request.

where λ_p and λ_w are defined as before. Thus, we can write the second-order approximation to the utility losses of the domestic representative consumer, resulting from inflation and output gap, as:

$$W = -\frac{1-\vartheta}{2}\sum_{t=0}^{\infty}\beta^t\left[\frac{1+\varphi}{1-\alpha}\tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p}\pi_{Ht}^2 + \frac{\epsilon_w(1-\alpha)}{\lambda_w}\pi_{wt}^2\right] \tag{A.12}$$

This is the welfare loss function used in the text.

B Calibration

Table 3: Calibration of parameters

Parameter	*Description*	*Value*
Preferences:		
β	Time discount factor	0.99
σ	Consumption risk aversion	1
φ	Labor supply elasticity	3
ϵ_p	Elasticity of substitution between individual goods	6
ϵ_w	Elasticity of substitution between labor types	6
ϑ	Openness of economy	0.4
η	Elasticity of substitution between domestic and foreign goods	1
γ	Elasticity of substitution between import countries	1
Calvo and technology:		
θ_w	Wage stickiness	3/4
θ_p	Price stickiness	3/4
α	Returns to scale in production	1/3
Policy rules:		
ϕ_H	Domestic inflation	1.5 or 0
ϕ_w	Wage inflation	1.5 or 0
ϕ_c	Composite inflation	1.5 or 0
ϕ_{CPI}	CPI inflation	1.5 or 0
ϕ_e	Exchange rate depreciation	1.5 or 0
$-\tau$	Payroll tax	15.7/100
Shock structure:		
ρ_a	Autocorrelation, technology	0.66
ρ_*	Autocorrelation, world output	0.86
σ_a	Standard deviation, technology shock	0.0071
σ_*	Standard deviation, world output shock	0.0078
$\rho_{a,*}$	Correlation between domestic technology shock and world shock	0.3
v	Monetary policy shock	0