An Intertemporal Approach to the Current Account

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Global imbalances are probably the most complex macroeconomic issue facing economists and policy makers

–Oliver Blanchard and Gian Maria Milesi-Feretti (2009)

Our plan for these two lectures (6 and 7)

- Establish an intertemporal (microfounded) equilibrium model suitable for a systematic and rigorous study of the current account.
- Use the model to address the following issues:
  - What determines CA surpluses and deficits?
  - How is the CA affected by fiscal policy?
  - Are CA deficits always bad?
  - Are CA deficits sustainable in the long run?
  - What determines the world interest rate?
  - How does the CA respond to shocks?
- Case studies:
  - Optimal fiscal policy rule in Norway (handlingsregelen).
  - The current sovereign debt crisis.
  - The last decade's global imbalances.

Readings

- Obstfeld, M. and K. Rogoff (1996): “Foundations of international macroeconomics”, ch. 1.1-1.3 and 2.1-2.3.
- Relevant material (and easy to read):
  - Bernanke, B. S. (2005): “The Global Saving Glut and the U.S. Current Account Deficit”, speech March 10, 2005.
  - Blanchard, O. and G. M. Milesi-Ferretti (2009): “Global Imbalances: In Midstream?”, IMF Staff Position Note 09/29. You can find it here.
  - Blanchard, O. and G. M. Milesi-Ferretti (2011): “(Why) Should Current Account Balances Be Reduced?”, IMF Staff Discussion Note 11/03. You can find it here.
  - IMF World Economic Outlook April 2012: “Growth Resuming, Dangers Remain”, ch. 1. You can find it here.

Outline

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1. Introduction – The current account
2. A two-period current account model for small open economies

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3. A two-period current account model for the world economy

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3. A two-period current account model for the world economy
4. An infinite-horizon current account model for small open economies

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What is the current account?

Definition

The current account represents . . .

- . . . the change in the value of net claims on the rest of the world.
- . . . the net increase in foreign asset holdings.

Current account =

+ Trade account

  Exports – imports of goods and services

+ Primary income account

  Payments for use of labor and financial resources

+ Secondary income account

  Redistribution (foreign aid, etc.)

The canonical current account equation

- Let B_{t+1} denote the value of an economy's net foreign assets at the end of period t (in what period is B_{t+1} a state variable?).
- The current account at time t, CA_t, is then defined as:

$$\begin{aligned} CA_t &= B_{t+1} - B_t \\ &= NX_t + rB_t \end{aligned}$$

- Net export:

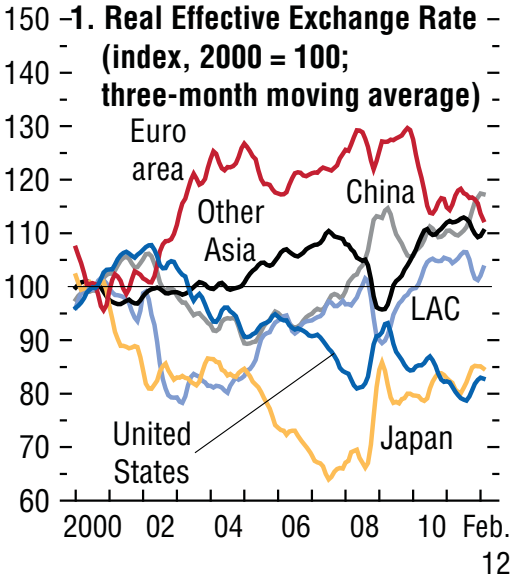
$$NX_t = Y_t - C_t - I_t - G_t$$

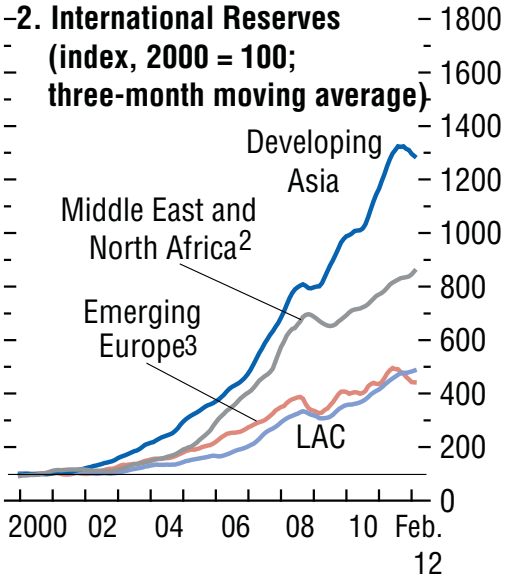
- Gross national income (GNI) is GDP plus income receipts from rest of the world:

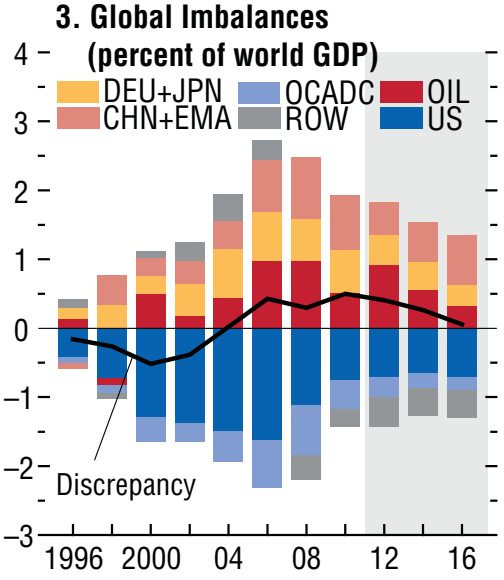
$$GNP_t = Y_t + rB_t$$

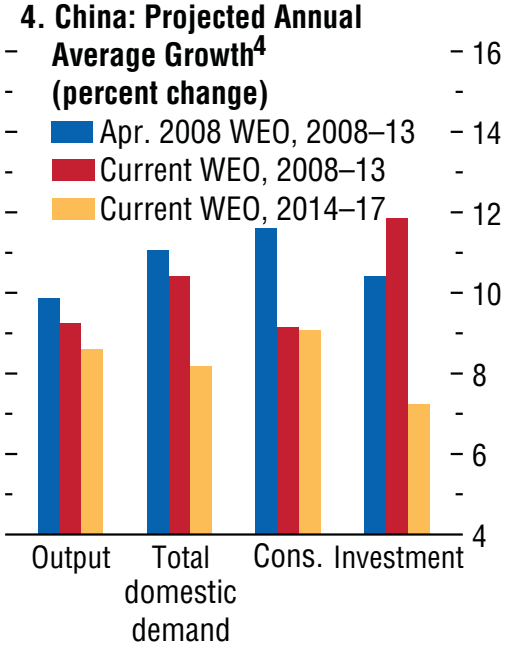
The current account:

$$CA_t = B_{t+1} - B_t = Y_t + rB_t - C_t - I_t - G_t \tag{1}$$









Sources: IMF, *International Financial Statistics;* and IMF staff estimates.
[1]CHN+EMA: China, Hong Kong SAR, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan Province of China, and Thailand; DEU+JPN: Germany and Japan; LAC: Latin America and the Caribbean; OCADC: Bulgaria, Croatia, Czech Republic, Estonia, Greece, Hungary, Ireland, Latvia, Lithuania, Poland, Portugal, Romania, Slovak Republic, Slovenia, Spain, Turkey, and United Kingdom; OIL: oil exporters; ROW: rest of the world; US: United States.
[2]Bahrain, Djibouti, Egypt, Islamic Republic of Iran, Jordan, Kuwait, Lebanon, Libya, Oman, Qatar, Saudi Arabia, Sudan, Syrian Arab Republic, United Arab Emirates, and Republic of Yemen.
[3]Bulgaria, Croatia, Hungary, Latvia, Lithuania, Poland, Romania, and Turkey.
[4]Variables in real terms. Cons. is total consumption.

Figure 1.3.1. China's Current Account and Components, 1971–2011

(Percent of GDP)

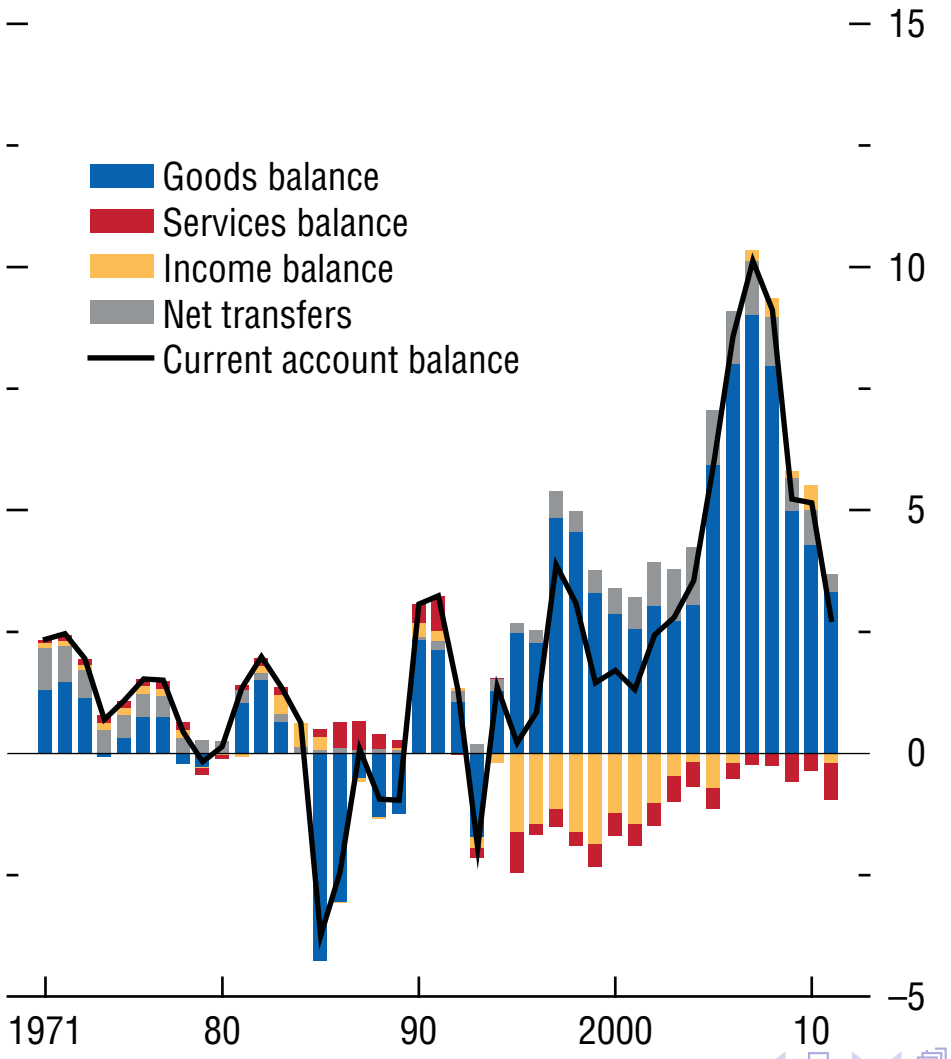


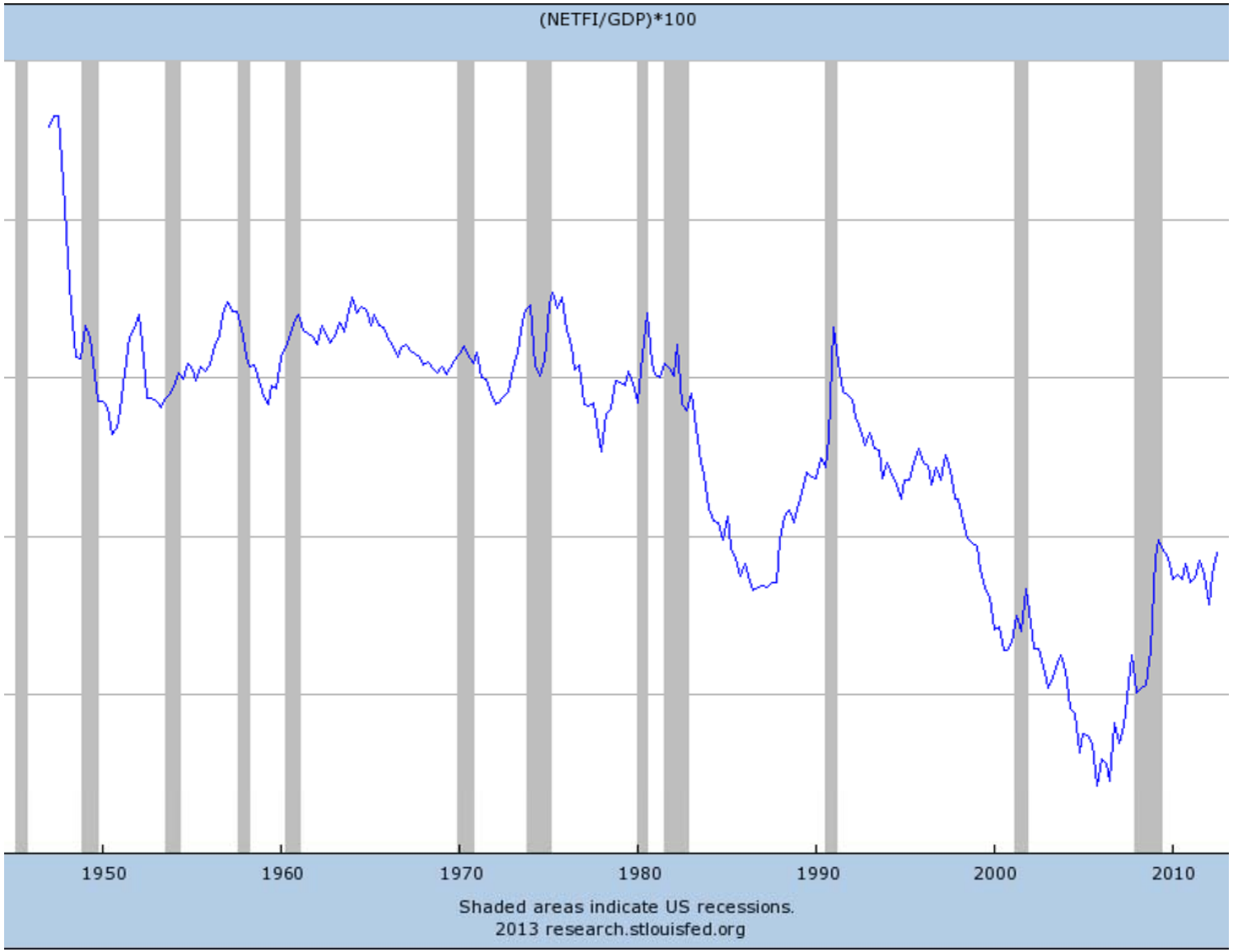
Table A10. Summary of Balances on Current Account

	2004	2005	2006	2007	2008	2009	2010	2011	Projections 2012	Projections 2013	Projections 2017
					Billions of U.S. Dollars						
Advanced Economies	**–207.8**	**–396.6**	**–433.5**	**–324.0**	**–496.4**	**–86.8**	**–85.4**	**–102.8**	**–157.6**	**–80.3**	**–255.8**
United States	–628.5	–745.8	–800.6	–710.3	–677.1	–376.6	–470.9	–473.4	–509.9	–499.0	–695.6
Euro Area[1,2]	121.6	51.9	53.7	45.6	–100.8	6.2	37.5	41.0	93.6	124.7	160.2
Japan	172.1	165.7	170.4	211.0	157.1	141.8	195.9	120.2	130.0	166.2	129.0
Other Advanced Economies[3]	127.0	131.7	143.0	129.8	124.5	141.8	152.2	209.5	128.7	127.7	150.6
Memorandum											
Newly Industrialized Asian Economies	87.1	83.2	99.0	128.3	86.5	123.8	137.2	134.8	127.8	132.8	161.5
Emerging and Developing Economies	**215.8**	**412.6**	**644.1**	**633.0**	**676.3**	**294.7**	**400.6**	**476.3**	**450.3**	**373.3**	**271.3**
Regional Groups											
Central and Eastern Europe	–55.1	–61.1	–89.0	–136.2	–159.9	–49.5	–81.8	–114.1	–108.7	–113.2	–160.1
Commonwealth of Independent States[4]	63.5	87.6	96.3	71.7	108.0	41.8	72.4	112.5	106.2	52.8	–62.4
Developing Asia	90.2	137.2	268.6	399.7	405.9	300.6	303.6	201.3	145.9	189.9	483.9
Latin America and the Caribbean	22.0	35.8	50.1	13.9	–32.1	–22.4	–55.2	–68.2	–107.2	–119.0	–181.2
Middle East and North Africa	103.1	213.9	287.5	271.4	353.3	52.0	186.2	366.0	439.7	399.0	272.5
Sub-Saharan Africa	–7.9	–0.7	30.6	12.5	1.2	–27.8	–24.6	–21.1	–25.6	–36.3	–81.5
Memorandum											
European Union	68.6	–4.3	–34.4	–66.9	–183.8	–15.9	–24.6	17.1	51.1	90.6	154.6
Analytical Groups											
By Source of Export Earnings											
Fuel	185.8	352.5	481.6	435.2	596.6	142.1	322.8	586.6	663.4	552.0	263.4
Nonfuel	30.1	60.1	162.5	197.7	79.7	152.7	77.8	–110.3	–213.1	–178.7	7.9
Of Which, Primary Products	–0.1	–0.9	10.0	7.8	–15.7	–2.1	–4.2	–18.2	–26.7	–23.9	–15.1
By External Financing Source											
Net Debtor Economies	–60.6	–90.3	–116.9	–227.3	–374.5	–190.0	–278.0	–343.1	–415.5	–427.7	–554.3
Of Which, Official Financing	–5.2	–6.0	–3.5	–5.4	–12.2	–9.4	–11.4	–13.4	–18.5	–16.1	–18.3
Net Debtor Economies by Debt-Servicing Experience											
Economies with Arrears and/or Rescheduling during 2006–10	–3.4	–6.7	–3.7	–13.8	–30.1	–24.7	–31.5	–38.9	–48.8	–48.1	–55.2
World[1]	**8.0**	**16.1**	**210.6**	**309.0**	**179.9**	**207.9**	**315.3**	**373.5**	**292.8**	**292.9**	**15.4**

Table A10. Summary of Balances on Current Account *(concluded)*

	2004	2005	2006	2007	2008	2009	2010	2011	Projections 2012	2013	2017
					Percent of GDP						
Advanced Economies	**–0.6**	**–1.1**	**–1.2**	**–0.8**	**–1.2**	**–0.2**	**–0.2**	**–0.2**	**–0.4**	**–0.2**	**–0.5**
United States	–5.3	–5.9	–6.0	–5.1	–4.7	–2.7	–3.2	–3.1	–3.3	–3.1	–3.5
Euro Area[1,2]	1.2	0.5	0.5	0.4	–0.7	0.0	0.3	0.3	0.7	1.0	1.1
Japan	3.7	3.6	3.9	4.8	3.2	2.8	3.6	2.0	2.2	2.7	1.9
Other Advanced Economies[3]	1.9	1.8	1.8	1.4	1.3	1.7	1.6	2.0	1.2	1.1	1.1
Memorandum											
Newly Industrialized Asian Economies	6.5	5.5	5.9	7.0	5.0	7.7	7.2	6.5	5.9	5.7	5.3
Emerging and Developing Economies	**2.4**	**3.8**	**5.0**	**4.0**	**3.5**	**1.6**	**1.9**	**1.9**	**1.7**	**1.3**	**0.7**
Regional Groups											
Central and Eastern Europe	–5.6	–5.2	–6.8	–8.3	–8.3	–3.1	–4.7	–6.0	–5.6	–5.5	–5.8
Commonwealth of Independent States[4]	8.2	8.7	7.4	4.2	5.0	2.5	3.7	4.6	4.0	1.7	–1.5
Developing Asia	2.6	3.4	5.6	6.6	5.5	3.8	3.2	1.8	1.2	1.4	2.5
Latin America and the Caribbean	1.0	1.3	1.6	0.4	–0.7	–0.6	–1.1	–1.2	–1.8	–2.0	–2.4
Middle East and North Africa	9.6	16.0	18.1	14.6	15.2	2.5	7.8	13.2	14.5	12.7	7.1
Sub-Saharan Africa	–1.5	–0.1	4.3	1.5	0.1	–3.1	–2.4	–1.8	–2.0	–2.6	–4.5
Memorandum											
European Union	0.5	0.0	–0.2	–0.4	–1.0	–0.1	–0.2	0.1	0.3	0.5	0.8
Analytical Groups											
By Source of Export Earnings											
Fuel	10.3	15.2	16.6	12.2	13.1	3.8	7.4	11.2	11.4	8.8	3.3
Nonfuel	0.4	0.7	1.6	1.6	0.5	1.1	0.4	–0.6	–1.0	–0.8	0.0
Of Which, Primary Products	0.0	–0.3	2.9	2.0	–3.5	–0.5	–0.8	–2.9	–3.9	–3.2	–1.5
By External Financing Source											
Net Debtor Economies	–1.2	–1.6	–1.8	–2.8	–4.0	–2.2	–2.7	–3.0	–3.5	–3.4	–3.3
Of Which, Official Financing	–2.6	–2.7	–1.4	–1.8	–3.4	–2.5	–2.8	–3.4	–4.3	–3.5	–2.9
Net Debtor Economies by Debt-Servicing Experience											
Net Debtor Economies with Arrears and/ or Rescheduling during 2006–10	–0.8	–1.4	–0.6	–2.0	–3.5	–3.0	–3.3	–3.6	–4.3	–4.0	–3.5
World[1]	**0.0**	**0.0**	**0.4**	**0.6**	**0.3**	**0.4**	**0.5**	**0.5**	**0.4**	**0.4**	**0.0**
Memorandum											
In Percent of Total World Current Account Transactions	0.0	0.1	0.7	0.9	0.5	0.7	0.8	0.9	0.6	0.6	0.0
In Percent of World GDP	0.0	0.0	0.4	0.6	0.3	0.4	0.5	0.5	0.4	0.4	0.0

The U.S. current account as %-share of GDP



Summary so far

- The current account is the value of claims on the rest of the world.
- Oil exporting countries, developing Asia, Japan and Germany run large surpluses. U.S. and Central and Eastern Europe run large deficits.
- Current account imbalances tend to decrease during recessions.

Outline

Environment

- Small open economy with both public and private sector. Utility maximizing households, profit maximizing firms.
- Agents can borrow and lend at a world market interest rate r (the interest rate is given).
- The economy exists for two periods, $t = 1, 2$.
- Representative household, population normalized to one.
- Perfect foresight (no uncertainty, no shocks).

Several of these assumptions will be relaxed later.

The representative household's utility

Lifetime utility for households:

$$U = u(C_1) + v(G_1) + \beta[u(C_2) + v(G_2)] \tag{2}$$

- $\beta \in (0, 1)$ is the discount factor (agents care less about the future).
- u is utility from private consumption and satisfies $u'(C) > 0$, $u''(C) < 0$, and $\lim_{C \to 0} u'(C) = \infty$.
- v is utility from public consumption and satisfies $v'(G) > 0$, $v''(G) < 0$, and $\lim_{G \to 0} v'(G) = \infty$.
- Important: Utility is separable in C and G, and G is determined exogenously by the government. This renders private agents' decisions (about C and I) independent of the level of G.

The public sector

The public sector collects lump sum taxes T_t and buys public consumption goods G_t. Disposable private income is $Y_t - T_t$.

- Public sector financial wealth after first period (assume $B_1^G = 0$):

$$T_1 - G_1 = B_2^G$$

- Public sector financial wealth after second period (why is $B_3^G = 0$?):

$$T_2 + (1 + r)B_2^G - G_2 = B_3^G = 0$$

- Combine to get public sector intertemporal budget constraint:

$$G_1 + \frac{G_2}{1 + r} = T_1 + \frac{T_2}{1 + r}$$

  LHS is present value of public sector consumption, RHS is present value of taxes.

Assume balanced government budgets ($B_t^G = 0$) from now on.

Output, capital and investment

The representative, competitive firm uses capital as input in production.

- Output:

$$Y_t = A_t F(K_t) \tag{3}$$

  $F'(K) > 0$, $F''(K) < 0$, and $F(0) = 0$.
- Assume initial capital is given and satisfies $K_1 > 0$.
- Law of motion for capital (assume zero depreciation):

$$K_{t+1} = K_t + I_t \tag{4}$$

- Remarks:
  a. $K_3 = 0$. Why?
  b. $I_2 = -K_2 = -(K_1 + I_1)$.

The household's intertemporal budget constraint

The representative household must decide how much to consume and invest.

- Financial wealth after first period (assume $B_1 = 0$):

$$Y_1 - T_1 - C_1 - I_1 = B_2$$

- Financial wealth after second period (why is $B_3 = 0$?):

$$Y_2 - T_2 + (1 + r)B_2 - C_2 - I_2 = B_3 = 0$$

- Combine the two and get the intertemporal budget constraint:

$$C_1 + I_1 + G_1 + \frac{C_2 + I_2 + G_2}{1 + r} = Y_1 + \frac{Y_2}{1 + r} \tag{5}$$

LHS is present value of private consumption, private investment, and public consumption. RHS is present value of output.

The current account

- Remember the accumulation equation (1) (CA equation):

$$CA_t = B_{t+1} - B_t = Y_t + rB_t - C_t - I_t - G_t$$

- Period budget constraints for households:

$$\text{Period 1:} \qquad B_2 + C_1 + I_1 + G_1 = Y_1$$
$$\text{Period 2:} \qquad C_2 + I_2 + G_2 = Y_2 + (1+r)B_2$$

- Implied current account:

$$\text{Period 1:} \qquad CA_1 = B_2 - B_1 = B_2 = Y_1 - C_1 - I_1 - G_1$$
$$\text{Period 2:} \qquad CA_2 = B_3 - B_2 = -B_2 = -CA_1$$

- Implied lifetime current account balance:

$$\sum_{t=1}^{2} CA_t = 0$$

The current account

- Insert (4) into the current account (1) and rewrite to express the change in domestic wealth as follows:

$$B_{t+1} + K_{t+1} - (B_t + K_t) = Y_t + rB_t - C_t - G_t$$

- Thus, we can define national saving:

$$S_t = Y_t + rB_t - C_t - G_t \tag{6}$$

- Combine (1) and (6) to get a new current account equation:

$$CA_t = S_t - I_t \tag{7}$$

Countries with less (more) savings than investments run current account deficits (surpluses).

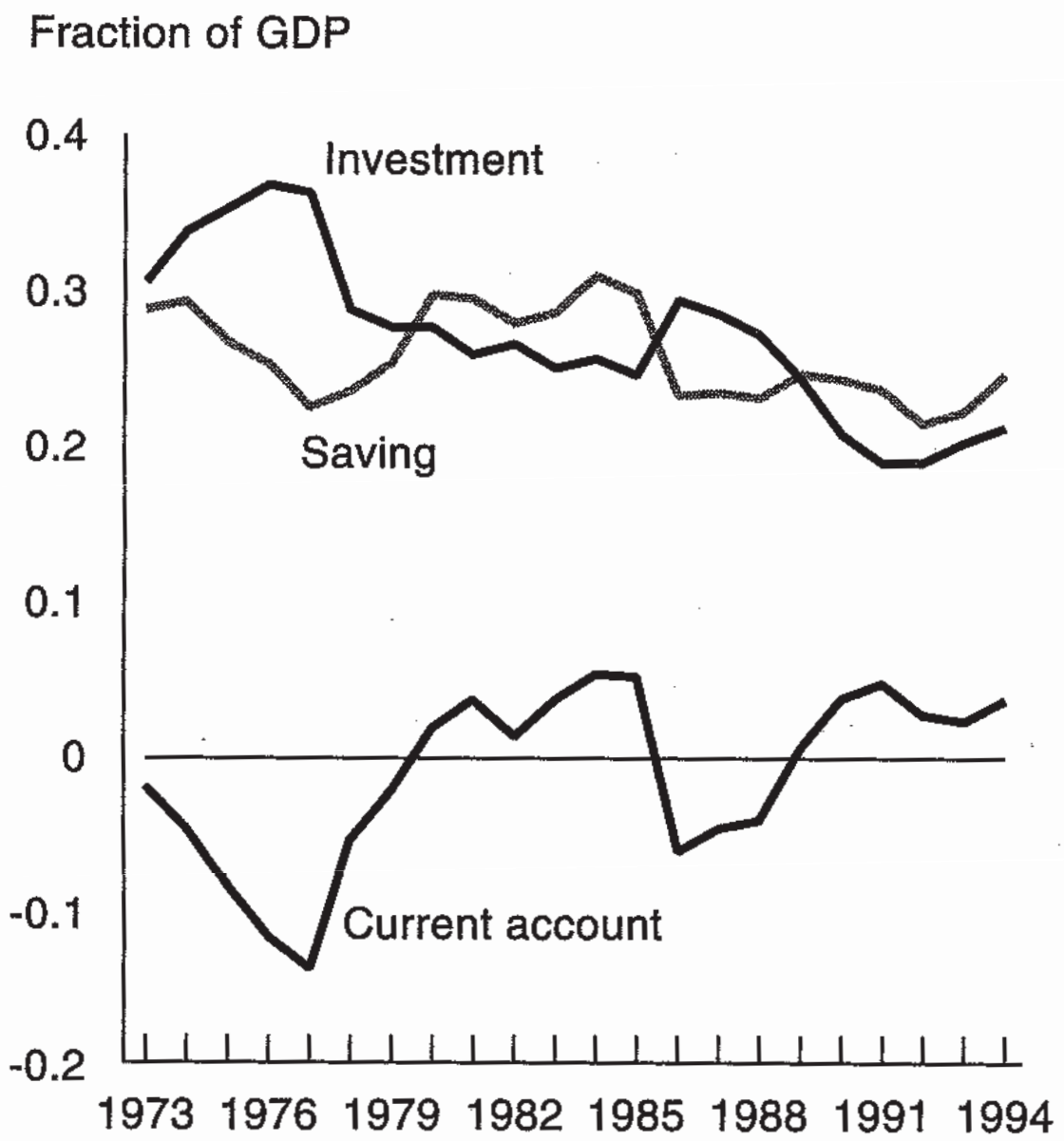


Figure 1.2
Norway's saving-investment balance, 1973–94. (*Source:* OECD)

The representative household's problem

The problem is to maximize lifetime utility subject to the intertemporal budget constraint. Our strategy is to make the problem unconstrained by inserting the constraint into the utility function:

- Solve the intertemporal budget constraint (5) for C_2:

$$C_2 = (1+r)\left[A_1 F(K_1) - C_1 - I_1 - G_1\right] + A_2 F(K_1 + I_1) - G_2 + K_1 + I_1 \tag{8}$$

- Insert into lifetime utility (2). Ignore $v(G)$ terms:

$$U = u(C_1) + \beta u\left((1+r)\left[A_1 F(K_1) - C_1 - I_1 - G_1\right] + A_2 F(K_1 + I_1) - G_2 + K_1 + I_1\right)$$

- First order conditions:

$$C_1 : \qquad \frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1+r} \tag{9}$$

$$I_1 : \qquad A_2 F'(K_2) = r \tag{10}$$

These are the familiar Euler equation and optimality condition with respect to capital!

The optimal consumption plan

Interpretation of optimality condition (9):

- $\frac{\beta u'(C_2)}{u'(C_1)}$ is household's marginal rate of substitution between present and future consumption.
- $\frac{1}{1+r}$ is the price of future consumption in terms of present consumption.
- Euler equation $\frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1+r}$ says it is optimal to set these two equal.
- Important special case is when $\beta(1+r) = 1$. Then $C_1 = C_2 = C$.
- Increasing consumption path if and only if $\beta(1+r) > 1$ (why?).
- Optimal consumption level can in principle be found by combining the Euler equation and the intertemporal budget constraint.

The optimal savings plan

Interpretation of optimality condition (10):

- Two ways of saving for the future in this model:
  a. Lending in international financial markets with constant returns r.
  b. Investing in productive capital at home with diminishing returns $A_2F'(K_2)$.
- The optimality condition $A_2F'(K_2) = r$ suggests investing at home until the marginal return on that investment is equal to returns in the international market.
- Remark: The desired level of capital is independent of preferences.

Gains from trade

One of the most important results from the model is that *intertemporal trade across borders is welfare improving for the small open economy*.

To talk about gains from trade we need 3 ingredients:

1. The production possibilities frontier (PPF).
2. The intertemporal budget constraint.
3. The household's indifference curves.

Gains from trade

1. The production possibility frontier (PPF):
   - Available technological possibilities for transforming period 1 consumption into period 2 consumption *in autarky*:

$$\begin{aligned} C_2 &= Y_2 + K_1 + S_1 - G_2 \\ &= A_2 F\left(Y_1 + K_1 - C_1 - G_1\right) + Y_1 + K_1 - C_1 - G_1 - G_2 \end{aligned}$$

   - Notice that $\frac{dC_2}{dC_1} = -A_2 F'\left(K_2\right) - 1 < 0$ and $\frac{d^2 C_2}{dC_1^2} = A_2 F''\left(K_2\right) < 0$.
2. The intertemporal budget constraint we have from before:

$$C_2 = Y_2 - I_2 - G_2 + (1 + r)\left(Y_1 - C_1 - I_1 - G_1\right)$$

3. The slope of the indifference curve is found using (9):

$$dU = u'(C_1)dC_1 + \beta u'(C_2)dC_2 = 0$$

$$\frac{dC_2}{dC_1} = -\frac{u'(C_1)}{\beta u'(C_2)} = -(1 + r) < 0$$

When $G_1 = G_2 = 0$:

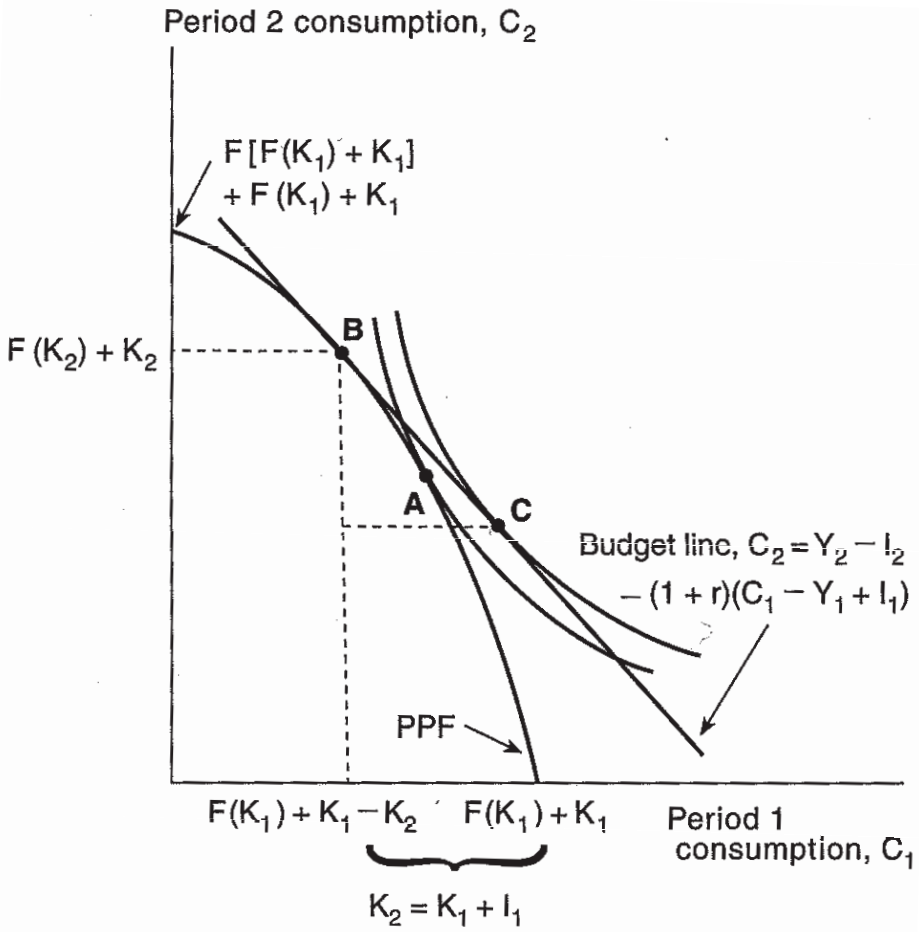


Figure 1.3
Investment and the current account

Gains from trade

The different allocations:

A Closed economy equilibrium with autarky rate r^A.

B Production when economy instead faces the world interest rate r. Maximizes the present value of domestic output net of investment.

C Open economy equilibrium. Gives the highest utility (from consumption) the small open economy can afford.

- The small open economy is better off with trade because the indifference curve that runs through C represents a higher utility level than the one running through A.
- There are gains from trade whenever $r \neq r^A$.
- There are gains from trade independently of a negative current account balance.

When $G_1 > 0$ and $G_2 = 0$:

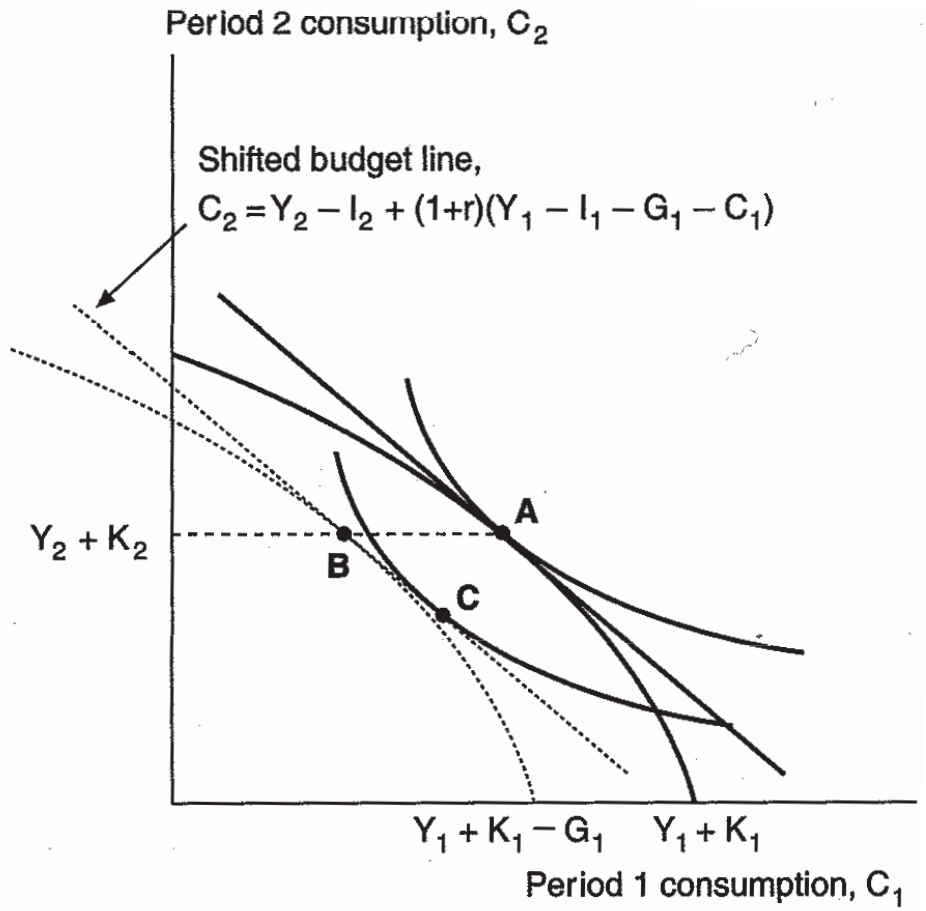


Figure 1.4
Government consumption and the current account

The role of government

- $G_1 > 0$ shifts the PPF horizontally to the *left* by the amount G_1, taking the economy's production from A to B.
- The disproportionally large decline in C_1 is dampened by consumption switching from the second to the first period (in order to reestablish the Euler equation).
- Thus, an economy with disproportionally high period 1 government consumption will (other things equal) tend to have a current account deficit in that period.
- If instead $G_2 > 0$ (and $G_1 = 0$), then the PPF shifts vertically *down* and a similar analysis applies. In particular, one expects (other things equal) a current account deficit in period 2 in this case.

Exercise

Suppose output is given by $Y_t = A_t K_t^{\alpha}$. The current account in period 1 is then $CA_1 = A_1 K_1^{\alpha} - C_1 - I_1$, where $I_1 = K_2 - K_1$.

1. Set up the representative household's problem and find the first order conditions with respect to C_1 and I_1.
2. With the first order conditions as points of departure, determine the Euler equation and the optimal level of K_2.
3. Suppose that A_1 increases. Use previous figures and results to discuss the effects of an increase in A_1 on CA_1.
4. Suppose the representative household suddenly (at the start of period 1) realizes that A_2 will be higher. Discuss the effects on CA_1.

Summary so far

- Today's absolute income level alone should not determine the current account.
- Rather, countries with high (low) current income, relative to future income, would tend to run surpluses (deficits).
- Current account deficits are not necessarily bad.
- Gains from intertemporal trade if $r \neq r_A$.
- Gains from trade because of consumption smoothing.
- Gains from trade with someone different from oneself.
- An important motivation for borrowing abroad is to finance investments. Norway in 1970's is one example.

Test question: does the model help explain ...

- Large surpluses among oil exporters?
- Big deficits in the U.S., big surpluses in China?

Outline

Motivation

- So far we have looked at a small open economy which, by being small, takes the world interest rate as given.
- Now it is time to study how the world interest is determined, and how it interacts with the current account.
- We will do this through the lenses of large economies (such as U.S. and E.U.).

Environment

- The world consists of two countries, Home and Foreign (*).
- Production in Home and Foreign:

$$Y_t = A_t F(K_t)$$
$$Y_t^* = A_t^* F^*(K_t^*)$$

- We ignore the public sector.
- By being large economies, Home and Foreign can now affect the world interest rate (it is no longer exogenous).
- Besides being large economies, everything is as in the small economy model, including equations (1)-(10).
- Foreign economy block has the same set of equations, but with a "*" on all variables and parameters.

World equilibrium

- World output must cover world demand for consumption and investment:

$$Y_1 + Y_1^* = C_1 + C_1^* + I_1 + I_1^* \tag{11}$$

- World saving must therefore be equal to world investment:

$$S_1 + S_1^* = Y_1 - C_1 + Y_1^* - C_1^* = I_1 + I_1^*$$

- Implied world current account:

$$CA_1 + CA_1^* = S_1 - I_1 + S_1^* - I_1^* = 0 \tag{12}$$

  Intuition: Whatever someone is lending, someone else must be borrowing.
- Walras' law: Consider an economy with n markets. If $n-1$ markets are in equilibrium, then the last market must also be in equilibrium. By Walras' law the world economy is in equilibrium if (12) holds.

World equilibrium

The current account, saving and investment in 2006 (percent of GDP)

Country	Current account	Saving	Investment
Germany	5.0	22.8	17.8
Japan	3.9	28.0	24.1
U.K.	-3.2	14.8	18.0
U.S.	-6.2	14.1	20.0
World	0	23.3	23.3

Saving and the world interest rate

- The world interest rate is determined by S_1, I_1, S_1^* and I_1^* (the two countries' net supply of intertemporal lending). But how?
- Since saving in Home is $S_1 = Y_1 - C_1$ (see (6)) and $Y_1 = A_1 F(K_1)$ is exogenous, the relationship between saving and the world interest rate is (similar for Foreign):

$$\frac{dS_1}{dr} = -\frac{dC_1}{dr}$$

Thus, we need to find $\frac{dC_1}{dr}$.

Savings schedule

- Substitute C_2 in (9) with (8):

$$u'(C_1) = \beta(1+r)u'((1+r)(A_1F(K_1) - C_1 - I_1) + A_2F(K_1 + I_1) + K_1 + I_1)$$

- Implicitly differentiating with respect to r:

$$\frac{dC_1}{dr} = \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)(A_1F(K_1) - C_1 - I_1)}{u''(C_1) + \beta(1+r)^2u''(C_2)}$$

- Assuming that period utility is isoelastic, i.e. $u(C_t) = \frac{C_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$:

$$\frac{dC_1}{dr} = \frac{(Y_1 - C_1 - I_1) - \sigma\frac{C_2}{1+r}}{1 + r + \frac{C_2}{C_1}} \tag{13}$$

- Conclusion: A rise in the world interest rate has an ambiguous effect on Home's period 1 consumption and saving.

The effects on saving and consumption of a rise in r

- Note that:

$$\frac{dS_1}{dr} = -\frac{dC_1}{dr} = -\frac{(Y_1 - C_1 - I_1) - \sigma\frac{C_2}{1+r}}{1 + r + \frac{C_2}{C_1}}$$
$$= -\frac{(S_1 - I_1) - \sigma\frac{C_2}{1+r}}{1 + r + \frac{C_2}{C_1}} = -\frac{CA_1 - \sigma\frac{C_2}{1+r}}{1 + r + \frac{C_2}{C_1}} \tag{14}$$

- If Home is a borrower in period 1, $CA_1 < 0$, then a rise in r unambiguously leads to more saving (and lower consumption).
- As r rises however, Home must eventually become a lender, i.e. $CA_1 > 0$. If S_1 is not that much higher than I_1, a rising r still increases saving and lower consumption.
- When r becomes sufficiently high, saving can actually drop and consumption increase.

Investment schedule

- Investment schedules are found by combining (10) and (4):

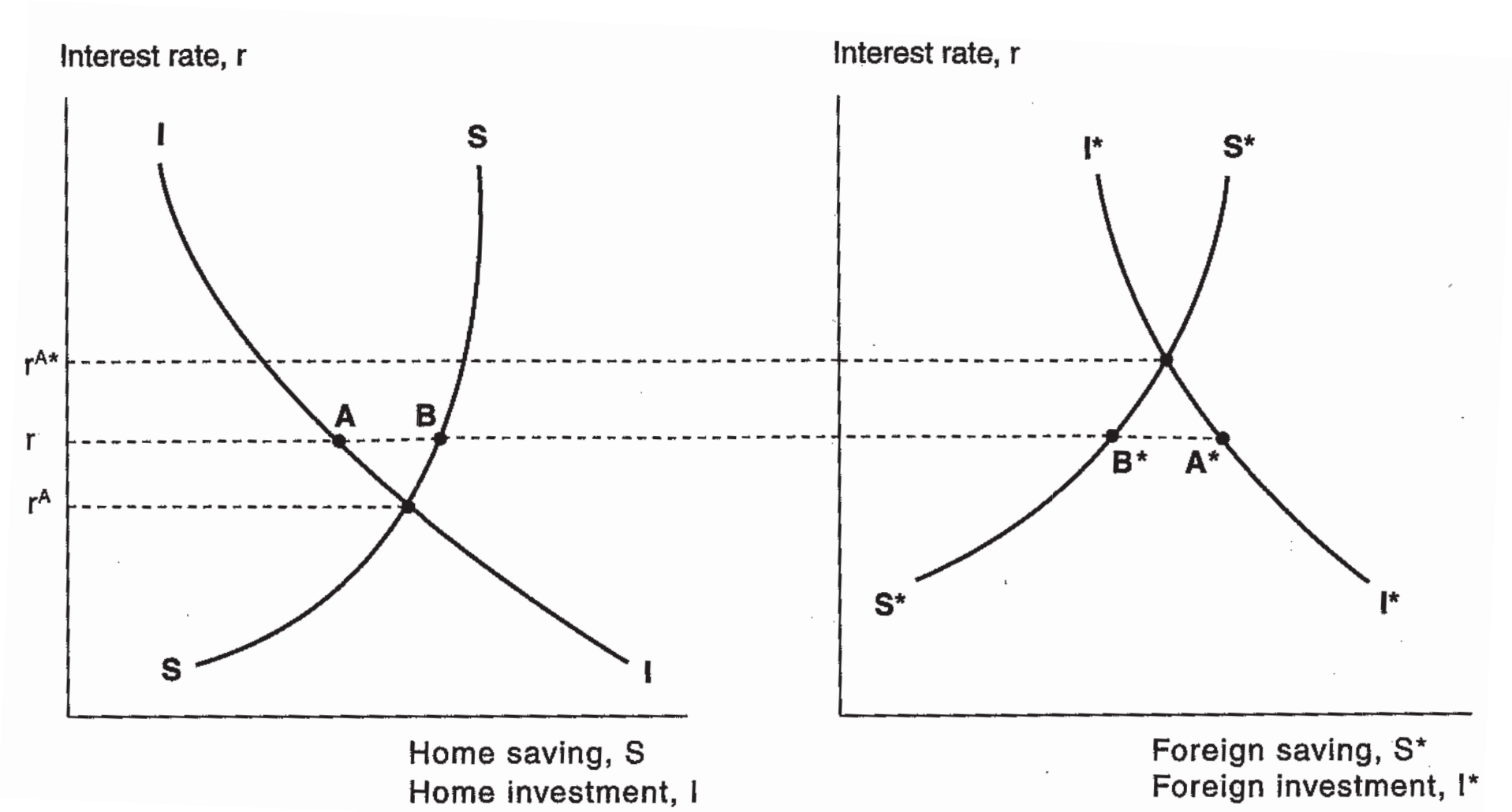
$$A_2 F(K_1 + I_1) = r \tag{15a}$$

$$A_2^* F^*(K_1^* + I_1^*) = r \tag{15b}$$

- A rise in r leads to lower I_1 and I_1^* (K_1 and K_1^* are predetermined as before).

The Metzler diagram

Savings and investments schedules, and global economy equilibrium:



Application I: An increase in Home's impatience

- Suppose households in Home become more impatient, reflected by a fall in β.
- Euler equation (9) tells us that lower β implies less period 2 consumption relative to period 1 consumption.
- *SS* in figure shifts to the left.
- *r* increases to restore equilibrium.
- Home saving and investment falls, Foreign investment falls, Foreign saving increases.

Application II: A rise in Home's future productivity

- Suppose A_2 suddenly is expected to be higher.
- The effect on the investment schedule given r is found from (15):

$$A_2 F'(K_1 + I_1) = r$$

$$F'(K_2) + A_2 F''(K_2) \left.\frac{dI_1}{dA_2}\right|_{dr=0} = 0$$

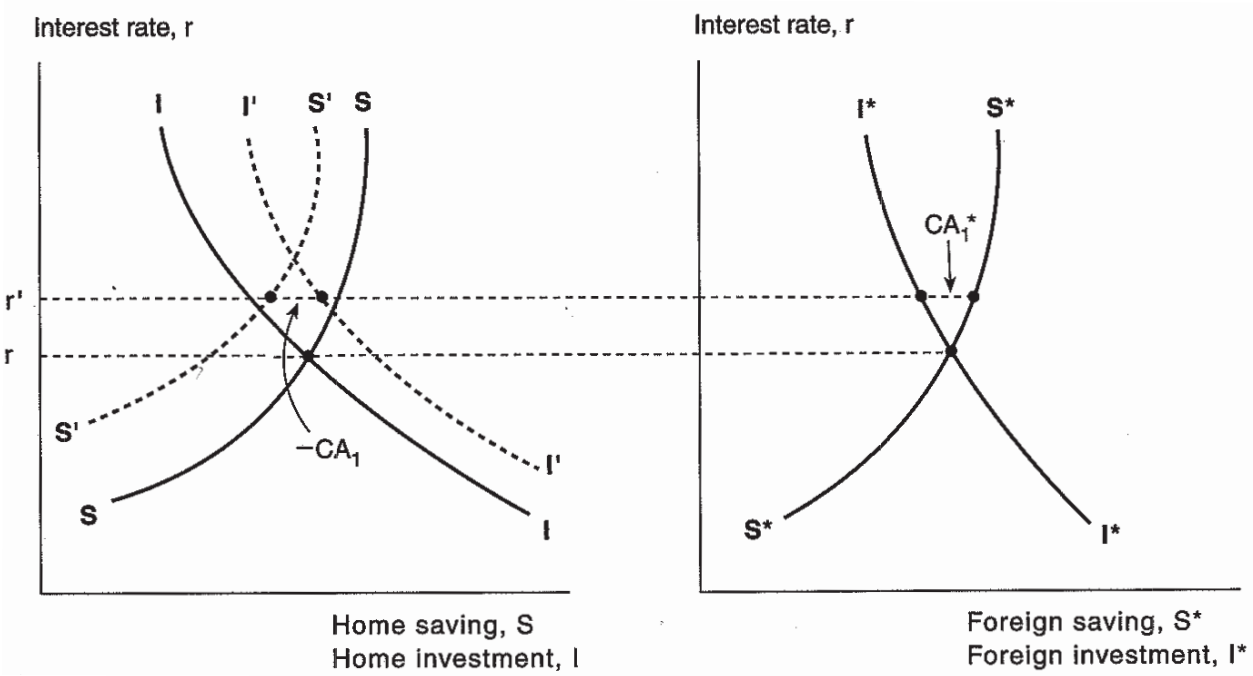
$$\left.\frac{dI_1}{dA_2}\right|_{dr=0} = -\frac{F'(K_2)}{A_2 F''(K_2)} > 0$$

  The investment schedule *II* shifts outwards.
- The effect on the savings schedule given r is ($S_1 = Y_1 - C_1$):

$$\left.\frac{dS_1}{dA_2}\right|_{dr=0} = -\left.\frac{dC_1}{dA_2}\right|_{dr=0} < 0$$

  The savings schedule *SS* shifts inwards.

Application II: A rise in Home's future productivity



- Assume initially balanced current accounts.
- Home's current account becomes negative.
- Foreign's current account becomes positive due to more saving and less investment.
- The world interest rate increases.

Summary

- Large economies affect the world interest rate.
- The global current account must be zero (what someone is lending, someone else must be borrowing).
- As in the small economy case, one runs a current account surplus (deficit) whenever current income is high (low) relative to future income.

This was the foundation. The next step is to establish a multi period model of the current account.

Outline

Motivation

Why abandon the two-period model?

- The world does not exist only for 2 periods.
- A two-period model may miss out on important dynamics.
- Difficult to do meaningful empirical analysis based on two-period results.

Additional important issues:

- What are the limits two a country's foreign debt (i.e. when is a country bankrupt)?
- Current account responses to permanent and temporary shocks.
- How does the analysis change in the case of uncertainty?

Setup

- Assume the economy exists infinitely. Suppose we are currently in period t. Let s be a generic time index.
- Lifetime (discounted) utility:

$$\begin{aligned} U_t &= u(C_t) + v(G_t) + \beta\left[u(C_{t+1}) + v(G_{t+1})\right] + \ldots \\ &= \sum_{s=t}^{\infty} \beta^{s-t}\left[u(C_s) + v(G_s)\right] \end{aligned} \tag{16}$$

- Output in period s:

$$Y_s = A_s F(K_s) \tag{17}$$

- Capital law of motion in period s:

$$K_{s+1} = K_s + I_s \tag{18}$$

- The current account in period s:

$$CA_s = B_{s+1} - B_s = Y_s + rB_s - C_s - I_s - G_s \tag{19}$$

The intertemporal budget constraint

- Rearrange the current account equation evaluated at $s = t$:

$$(1 + r)\, B_t = C_t + I_t + G_t - Y_t + B_{t+1}$$

- Forward by one period and solve for B_{t+1}:

$$B_{t+1} = \frac{C_{t+1} + I_{t+1} + G_{t+1} - Y_{t+1}}{1 + r} + \frac{B_{t+2}}{1 + r}$$

- When the two above are combined, we can expand the series using forward substitution until $s = T$:

$$\begin{aligned}(1 + r)B_t &= C_t + I_t + G_t - Y_t + \frac{C_{t+1} + I_{t+1} + G_{t+1} - Y_{t+1}}{1 + r} + \frac{B_{t+2}}{1 + r} \\ &= \ldots \\ &= \sum_{s=t}^{T} \left(\frac{1}{1 + r}\right)^{s-t} (C_s + I_s + G_s - Y_s) + \left(\frac{1}{1 + r}\right)^{T} B_{t+T+1}\end{aligned}$$

A transversality condition

- Let us impose the following (referred to as a transversality condition):

$$\lim_{T\to\infty}\left(\frac{1}{1+r}\right)^{T} B_{t+T+1} = 0$$

- Interpretation:

  If "< 0": Must mean $B_{t+T+1} < 0$ and growing faster than the interest rate. Creditors will not allow this forever, i.e. limit must be ≥ 0.

  If "> 0": Must mean $B_{t+T+1} > 0$ and growing faster than the interest rate. Country is providing resources to foreigners without getting them back. Not utility maximizing behavior, i.e. limit must be ≤ 0.

The intertemporal budget constraint

- Assume the transversality condition holds. Rearrange the budget constraint and take the limit as $s \to \infty$:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (C_s + I_s) = (1+r) B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} (Y_s - G_s) \quad (20)$$

- This is the intertemporal budget constraint in the infinite horizon model. LHS is present value of consumption and investment. RHS is present private value of income.

The representative household's problem

The problem is to maximize lifetime utility subject to the intertemporal budget constraint and the current account. Our strategy is to:

- Solve the current account (19) for C_s to get:

$$C_s = Y_s + (1 + r) B_s - B_{s+1} - I_s - G_s \tag{21}$$

- Insert this into the lifetime utility function (16) and find the first order conditions with respect to B_{s+1} and K_{s+1}.
- Combine these optimality conditions with the intertemporal budget constraint (20) to characterize equilibrium in the model.

Next we show how to do this!

The representative household's problem

- Maximize the following with respect to B_{s+1} and K_{s+1} (ignore terms with $v(G_s)$):

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u\left(A_s F\left(K_s\right) + (1+r) B_s - B_{s+1} - \left(K_{s+1} - K_s\right) - G_s\right)$$

- First order conditions:

$$B_{s+1}: \qquad \frac{\beta u'(C_{s+1})}{u'(C_s)} = \frac{1}{1+r} \tag{22}$$

$$K_{s+1}: \qquad A_{s+1} F'(K_{s+1}) = r \tag{23}$$

- These are the familiar Euler equation and optimality condition with respect to capital, i.e. the infinite-horizon versions of (9) and (10)!

The optimal consumption plan

- Define lifetime wealth (in period t) net of investment and government consumption:

$$W_t = (1+r)\,B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - I_s - G_s) \tag{24}$$

- Combine (24) with intertemporal budget constraint (20):

$$\begin{aligned}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s &= (1+r)\,B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - I_s - G_s)\\ &= W_t \end{aligned} \tag{25}$$

Case I: Optimal consumption plan if $\beta(1+r)=1$

- Euler in this case implies $C_t = C_{t+1} = C_{t+2} = ... \equiv \bar{C}$. Use that insight to solve (25) for $\bar{C}$. The result is:

$$\begin{aligned}\bar{C} &= \frac{r}{1+r}\left[(1+r)\,B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - I_s - G_s)\right] \\ &= \frac{r}{1+r}W_t \end{aligned} \tag{26}$$

- Optimal consumption is annuity value of net total wealth.
- How much would be consumed of current financial wealth?
- What country is supposed to consume exactly this amount of its financial wealth?

Case II: Optimal consumption plan if utility is isoelastic

- Suppose utility is isoelastic (we relax the assumption that $\beta(1+r)=1$):

$$u(C_s) = \frac{C_s^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

- Euler equation (22) gives us the optimal consumption growth rate:

$$C_s^{-\frac{1}{\sigma}} = \beta(1+r)\, C_{s+1}^{-\frac{1}{\sigma}}$$
$$g_C = \frac{C_{s+1}-C_s}{C_s} = \beta^{\sigma}(1+r)^{\sigma} - 1$$

- Optimal consumption growth is constant (as long as r is constant). It is positive if $\beta(1+r) > 1$. It is negative if $\beta(1+r) < 1$. It is high in absolute value if σ is high. Can you interpret these results? Have you seen them before?

Case II: Optimal consumption plan if utility is isoelastic

- Rewrite Euler equation:

$$C_{t+1} = \beta^{\sigma}(1+r)^{\sigma} C_t = (1+g_C)\, C_t$$

- Forward substitution s periods ahead gives $C_s = (1+g_C)^{s-t} C_t$.
- Substitute into (25) and assume $r > g_C$. Optimal consumption is:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} C_s = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (1+g_C)^{s-t} C_t = W_t$$

$$C_t = \frac{r-g_C}{1+r} W_t \tag{27}$$

- Compare with (26). If $g_C > 0$, then consume less than annuity value to obtain growth in consumption over time.

Outline

Application I: The optimal fiscal rule in Norway

Motivation:

- Norway has (very) large oil revenues. Some background here.
- The rule (from 2001): Invest oil income in a fund abroad. Spend the expected real return of the fund each year (estimated to be 4% annually in 2001).
- Big debate in Norway about the optimal spending plan.
  - The government wants to keep the current fiscal rule (spend 4% of the fund each year).
  - But that stand is criticized by many economists. See for instance here.
- What does our model have to say about the optimal fiscal rule in Norway?

Application I: The optimal fiscal rule in Norway

The optimal spending plan:

- Norway invests oil wealth abroad and expects to get it back with returns. In our model it works as increased foreign claims B_t.
- Case 1: $\beta = \frac{1}{1+r}$
  - From (26): Optimal spending rule is r, i.e. the real return. Consistent with policy!
  - But: Real net returns (after management costs and inflation) the last ten years has been 2.4% on average, not 4% (Norges Bank årsrapport 2011).
- Case 2: Isoelastic utility
  - From (27): Optimal spending rule is $r - g_C$.
  - Average g_C is 2.5% the last ten years (when measured as per capita household consumption growth), further implying that a fiscal rule of 4% may be too high.

Application II: The risk of sovereign default within EMU

Motivation:

- Considerable amounts of research on sovereign debt (country level debt) these days.
- The financial crisis evolved into a sovereign debt crisis.
- The European sovereign debt crisis:

  > *From late 2009, fears of sovereign defaults developed among investors as a result of the rising of government debt levels around the world, together with a wave of downgrading of government debt in some European states.*

- What can be said about the issue when using our model?

Application II: The risk of sovereign default within EMU

- Rearrange the budget constraint (20) to get an expression for the sum of trade balances NX_s:

$$-(1+r)\,B_t = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - C_s - I_s - G_s)$$
$$= \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} NX_s$$

- Interpretation: Current net foreign debt is equal to present value of trade surpluses.
- Suppose $Y_{s+1} = (1+g)\,Y_s$. Then the debt-output ratio $\frac{B_s}{Y_s}$ is stabilized only if debt grows at the same rate, $B_{s+1} = (1+g)\,B_s$.

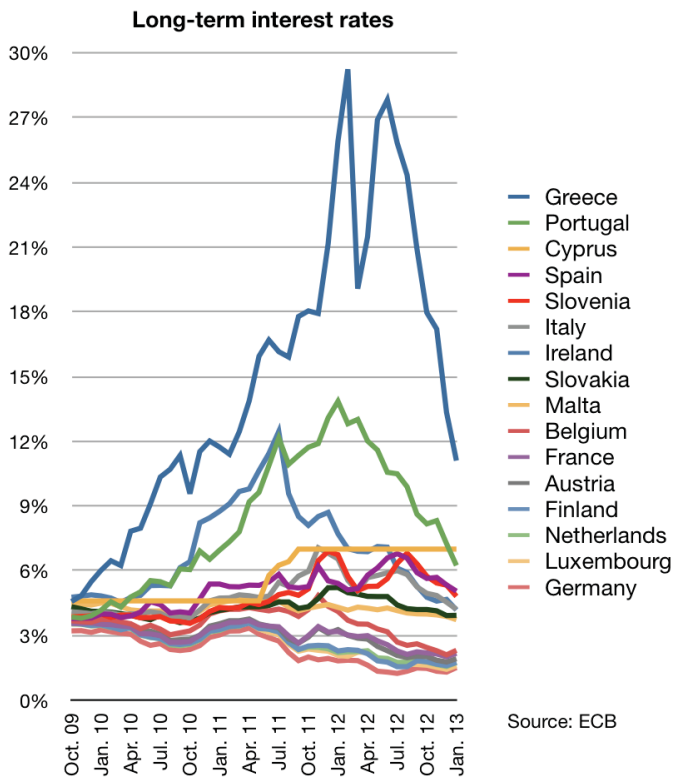
Application II: The risk of sovereign default within EMU

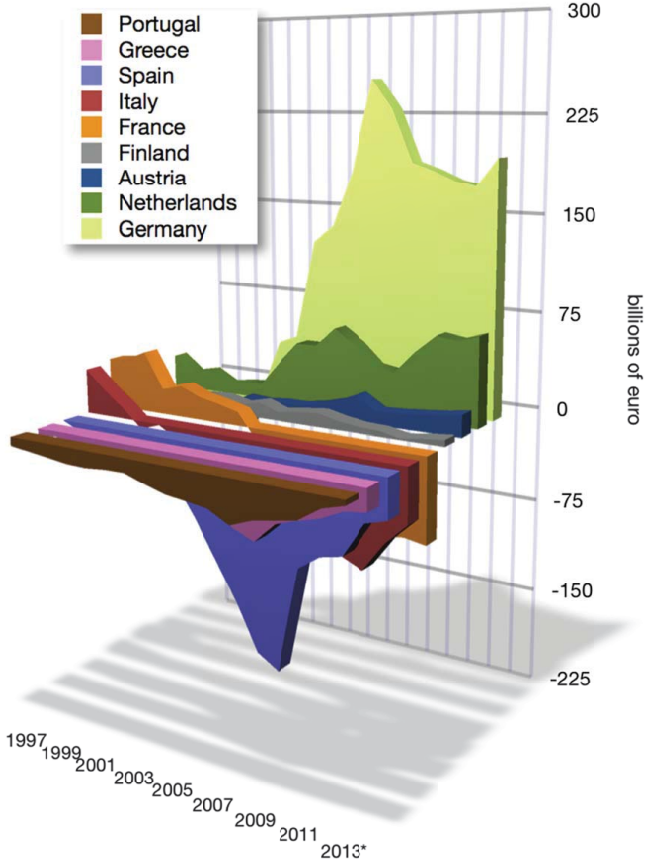
- Let us see what this implies for the trade balance. From the current account:

$$CA_s = B_{s+1} - B_s = gB_s = rB_s + NX_s$$
$$\frac{NX_s}{Y_s} = \frac{-(r-g)B_s}{Y_s} \tag{28}$$

  Equation (28) must hold if a country is to keep the ratio of debt to GDP fixed.
- Assume $r > g$ holds (it typically does in countries with large debt).
- Suppose a country is a net debtor (Greece), i.e. $B_s < 0$. Then the country must run a trade surplus equal to $\frac{-(r-g)B_s}{Y_s}$ in order to stabilize the ratio of debt to GDP.
- The ratio $\frac{-(r-g)B_s}{Y_s}$ measures the burden of foreign debt in an economy. The higher this ratio, the greater the likelihood that the debt is unsustainable.

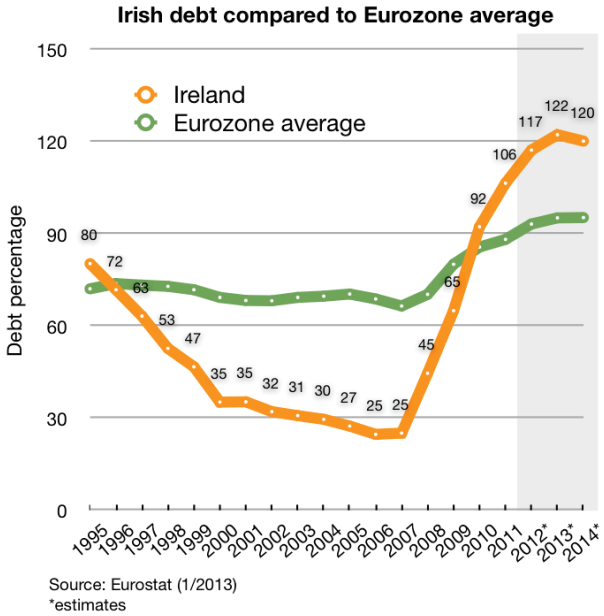
Application II: The risk of sovereign default within EMU

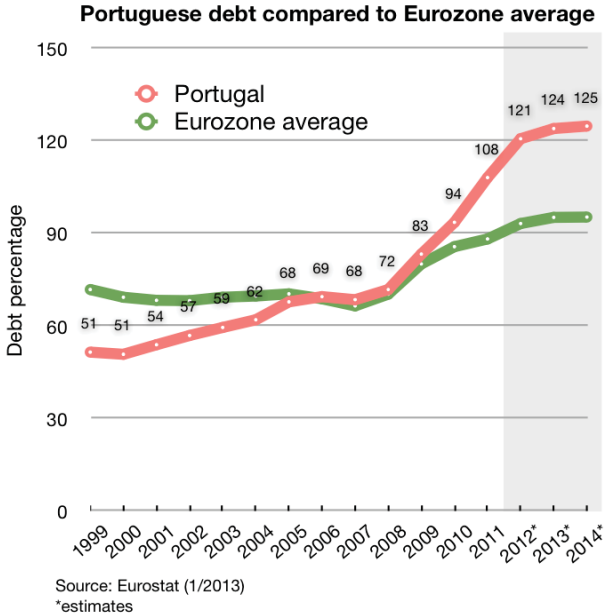


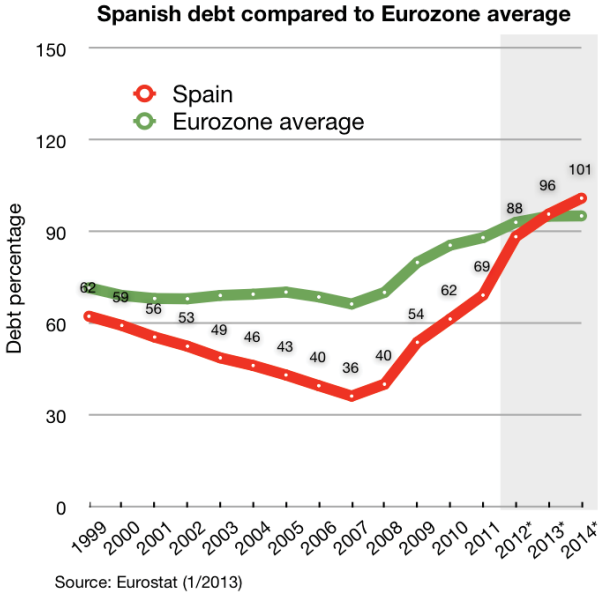


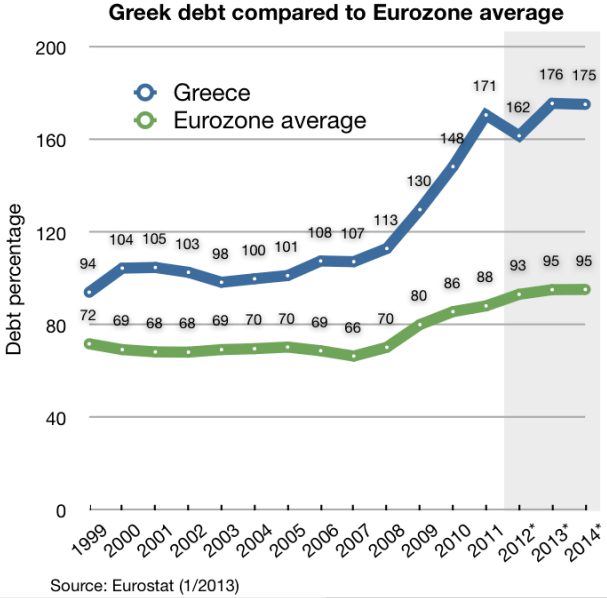
Trade imbalances

Application II: The risk of sovereign default within EMU









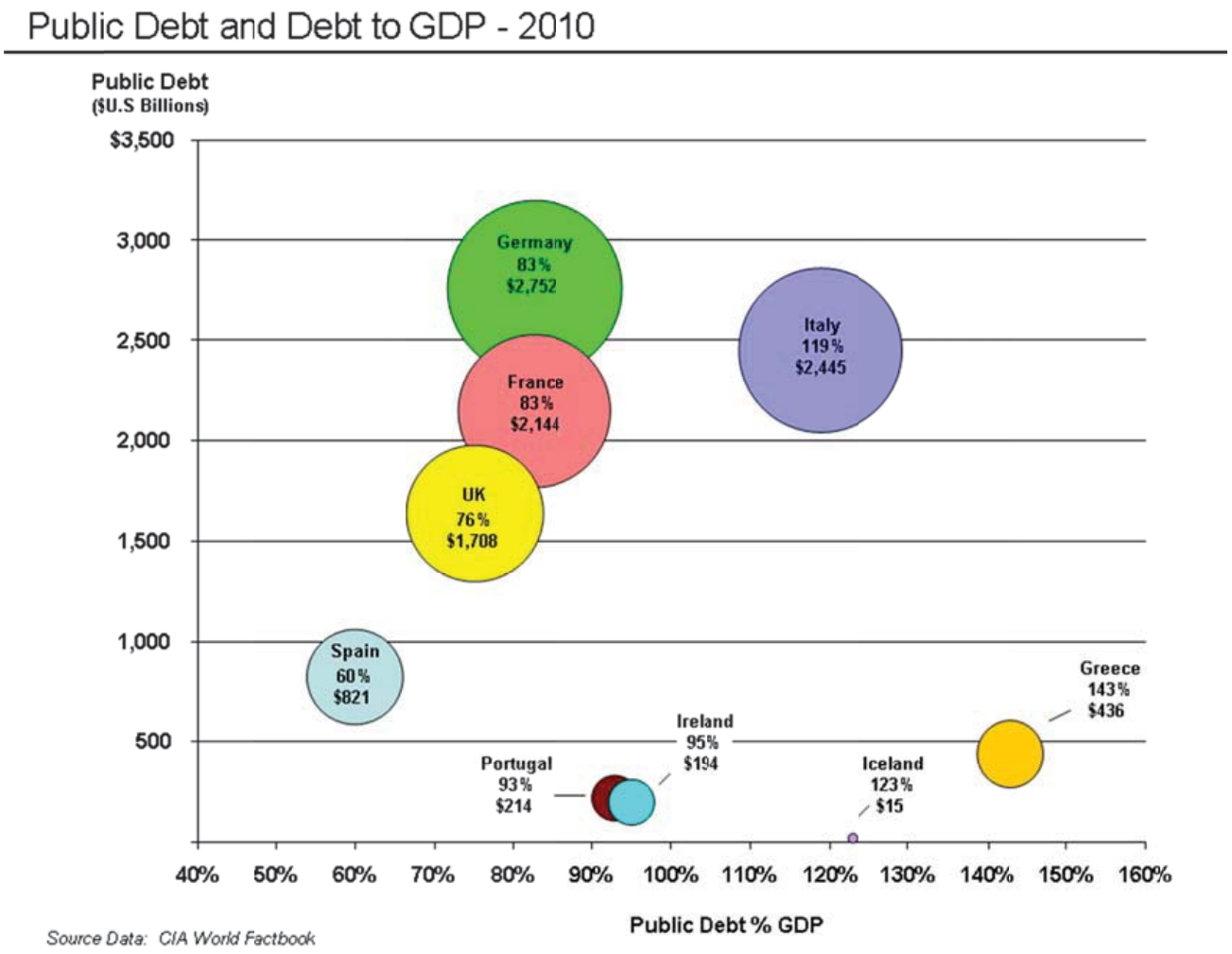
Application II: The risk of sovereign default within EMU

The debt burden in Greece:

- Debate in media: Will Greece default?
- Critical to stabilize (and revert) Greek debt in order to get the situation under control.
- Greece this winter: r around 11%, g approximately -5%, B_s/Y_s about -160%. Plug numbers into RHS of (28).
- Implies that Greece needs a trade surplus equal to 25.6% of GDP to stabilize its debt.
- However, NX_s/Y_s (LHS of (28)) is approximately -8% (rough estimate).
- Not sustainable!

Application II: The risk of sovereign default within EMU

However, Italy poses a much bigger risk than Greece:



Outline

The annuity value of a variable

- For a constant interest rate r, define the permanent level of a variable X on date t by:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\tilde{X}_t=\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$$

- Solve for $\tilde{X}_t$:

$$\tilde{X}_t=\frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s \tag{29}$$

- Interpretation: $\tilde{X}_t$ is the annuity value of $\{X_s\}_{s=t}^{\infty}$ at the prevailing interest rate, i.e. the hypothetical constant level of the variable with the same present value as the variable itself.
- Straight forward to see that $\frac{1+r}{r}\tilde{X}_t=\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$.

The annuity value of a variable

- Using the same transformation as in (29), lifetime wealth (24) can be written:

$$W_t = (1+r)\,B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - I_s - G_s)$$
$$= (1+r)\,B_t + \frac{1+r}{r}\left(\tilde{Y}_t - \tilde{I}_t - \tilde{G}_t\right), \qquad (30)$$

where $\frac{1+r}{r}\tilde{Y}_t = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Y_s$, $\frac{1+r}{r}\tilde{I}_t = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} I_s$, and $\frac{1+r}{r}\tilde{G}_t = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} G_s$.

- Equation (30) is useful when we study current account dynamics in the infinite horizon model.

Case I: Current account dynamics if $\beta(1+r)=1$

- Insert (26) and (30) into (19). The current account becomes:

$$\begin{aligned} CA_t &= Y_t + rB_t - C_t - I_t - G_t \\ &= \left(Y_t - \tilde{Y}_t\right) - \left(I_t - \tilde{I}_t\right) - \left(G_t - \tilde{G}_t\right) \end{aligned} \tag{31}$$

- Vital predictions:
  - Output above its permanent level contributes to a current account surplus due to consumption smoothing.
  - Similarly, people use foreign borrowing to cushion their consumption in the face of unusually high investment needs (Norway in the 1970's), rather than financing profitable opportunities entirely out of domestic savings.
  - Finally, abnormal high government spending needs have the same effect as abnormally low output.

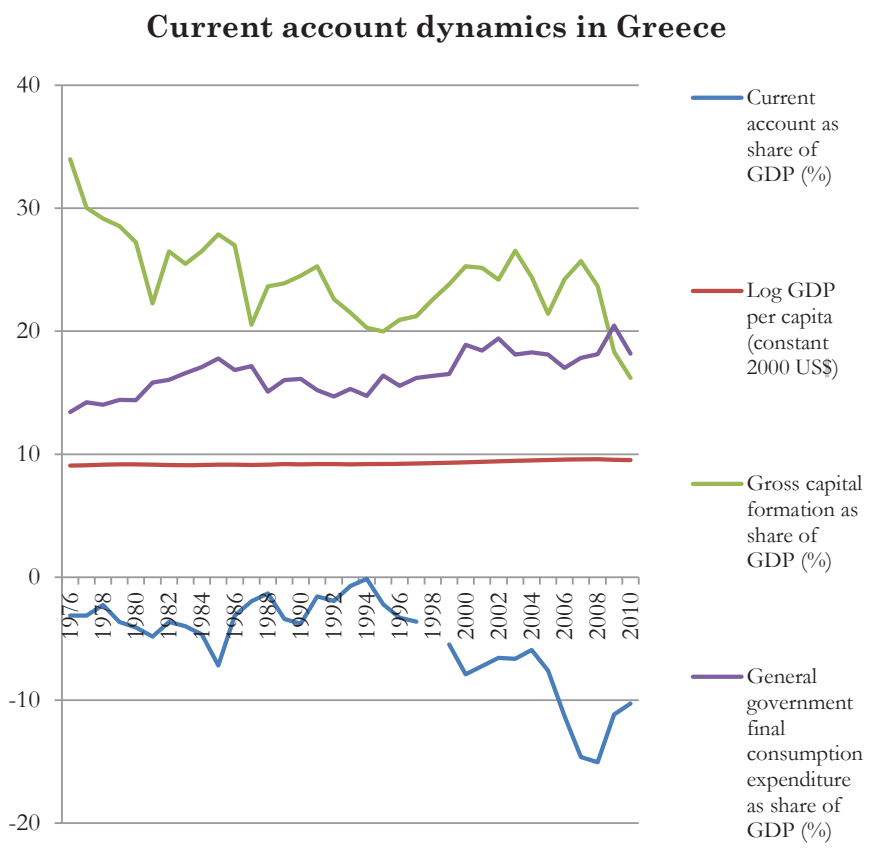
Case II: Current account dynamics if utility is isoelastic

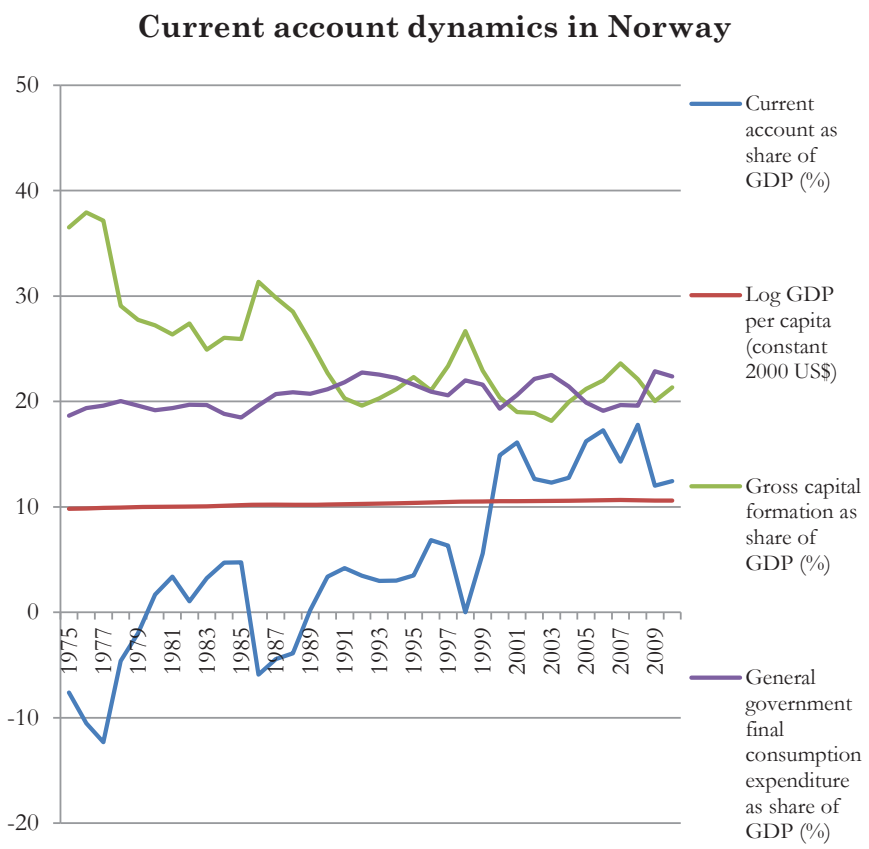
- Insert (27) and (30) into (19). The current account becomes:

$$\begin{aligned} CA_t &= Y_t + rB_t - C_t - I_t - G_t \\ &= \left(Y_t - \tilde{Y}_t\right) - \left(I_t - \tilde{I}_t\right) - \left(G_t - \tilde{G}_t\right) + \frac{g_C}{1+r} W_t \end{aligned} \tag{32}$$

- What distinguishes (32) from (31) is the presence of a consumption tilt factor.
- The tilting motive is determined by the sign of g_C i.e. the size of $\beta(1+r)$.
- Can you interpret this?

Current account data revisited (World Bank data)





A stochastic current account model

Motivation:

- So far we have endowed people with perfect foresight. It is time to impose a weaker assumption, namely that individuals have rational expectations.
- People do not perfectly foresee random economic events in the future.
- Future levels of output, investment and government spending are stochastic.
- Decisions today must be based on informed guesses. As new information comes along, people revise previous plans.
- Rational expectations:

> *"Mathematical conditional expectations based on an accurate model of the economy's structure and all the information about the current economic variables that are available to the individual."*

A stochastic current account model

Preliminaries – Rational Expectations Hypothesis (REH)

- Consider a stochastic time series $\{X_s\}_{s=t}^{\infty}$. REH states that individual's subjective expectations are equal to the conditional mathematical expectation of that variable. For instance, the subjective expectation at time t of the variable at time $t+1$ is $E_t X_{t+1}$.
- Rational expectations forecast error:

$$\epsilon_{t+1} = X_{t+1} - E_t X_{t+1}$$

- According to REH, then:

$$E_t(\epsilon_{t+1}) = E_t(X_{t+1} - E_t X_{t+1}) = E_t X_{t+1} - E_t X_{t+1} = 0$$

ϵ_{t+1} has mean zero and is uncorrelated with the information available at time t.

A stochastic current account model

Environment:

- Small open economy where future output levels follow a stochastic process.
- Only riskless asset is a bond which pays a constant real interest rate r.
- *Expected lifetime utility* at time t is:

$$E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[u\left(C_s\right) + v(G_s) \right] \right\} \tag{33}$$

- Output, capital law of motion, the current account and the intertemporal budget constraint are given as before.
- Future levels are stochastic, however. Equations therefore only holds in expected terms.

A stochastic current account model

Optimality conditions:

- When the current account (19) is solved for C_s and the result is inserted into the utility function (ignore $v(G_s)$):

$$E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u \left(A_s F(K_s) + (1+r) B_s - B_{s+1} - (K_{s+1} - K_s) - G_s \right) \right\}$$

- First order conditions:

$$B_{s+1}: \qquad \frac{\beta E_t u'(C_{s+1})}{u'(C_s)} = \frac{1}{1+r} \tag{34}$$

$$K_{s+1}: \qquad E_t A_{s+1} F'(K_{s+1}) = r \tag{35}$$

These are the rational expectations equivalents to (22) and (23).

A stochastic current account model

A certainty equivalence result:

- Let us look at a special case of the stochastic model just introduced. Suppose period utility is $u(C_t) = C_t - \frac{\gamma}{2} C_t^2$.
- Assume $\beta(1+r) = 1$. The stochastic Euler equation at time t:

$$C_t = E_t C_{t+1} \tag{36}$$

- Thus, consumption follows a random walk in this case ($E_t C_s = \ldots = E_t C_{t+1} = C_t$).
- Insert (36) into (20) and combine with (24). The result is:

$$\begin{aligned} C_t &= \frac{r}{1+r}\left[(1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} E_t(Y_s - I_s - G_s)\right] \\ &= \frac{r}{1+r} E_t W_t \end{aligned}$$

- Certainty equivalence: Decision making as if expected values were certain to be realized.

A stochastic current account model

- Make some simplifying assumptions:
  - Endowment economy (no investment) and no public sector.
  - Output gap deviations from steady state $\bar{Y}$ follow an AR(1) process:

$$Y_{t+1} - \bar{Y} = \rho\left(Y_t - \bar{Y}\right) + \epsilon_{t+1}, \quad 0 \leq \rho \leq 1 \tag{37}$$
$$E_t\left(Y_s - \bar{Y}\right) = \rho^{s-t}\left(Y_t - \bar{Y}\right), \quad s > t$$

- Implied consumption and current account functions (details in class):

$$C_t = rB_t + \bar{Y} + \frac{\rho r}{1 + r - \rho}\left(Y_{t-1} - \bar{Y}\right) + \frac{r}{1 + r - \rho}\epsilon_t \tag{38}$$

$$CA_t = \rho\frac{1 - \rho}{1 + r - \rho}\left(Y_{t-1} - \bar{Y}\right) + \frac{1 - \rho}{1 + r - \rho}\epsilon_t \tag{39}$$

  Notice that the CA is zero (always) if $\rho = 1$ (random walk).
- Model is closed with CA identity:

$$B_{t+1} = CA_t + B_t \tag{40}$$

A simple simulation exercise

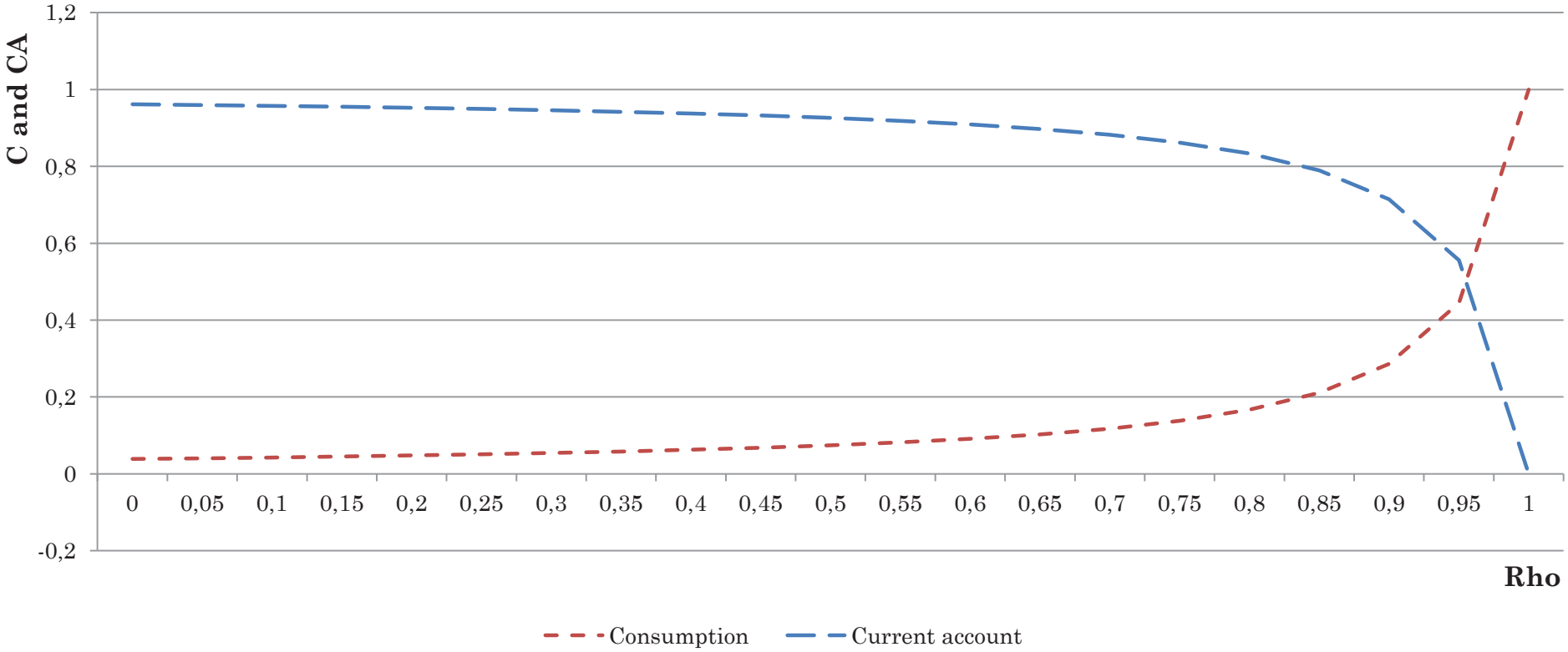
Now we have a 4-equation system (37)-(40) in 4 variables $(Y_t - \bar{Y})$, CA_t, C_t and B_{t+1}. This system constitutes a simple rational expectations CA model.

Let us simulate a unitary shock to output using equations above.

- Calibration: $\beta(1+r) = 1$ (implies $\beta \approx 0.96$), $\bar{Y} = 0$, $B_1 = 0$, $\epsilon_1 = 1$.
- You can find an Excel sheet with the simulations on It's Learning. Feel free to play around with different calibrations!

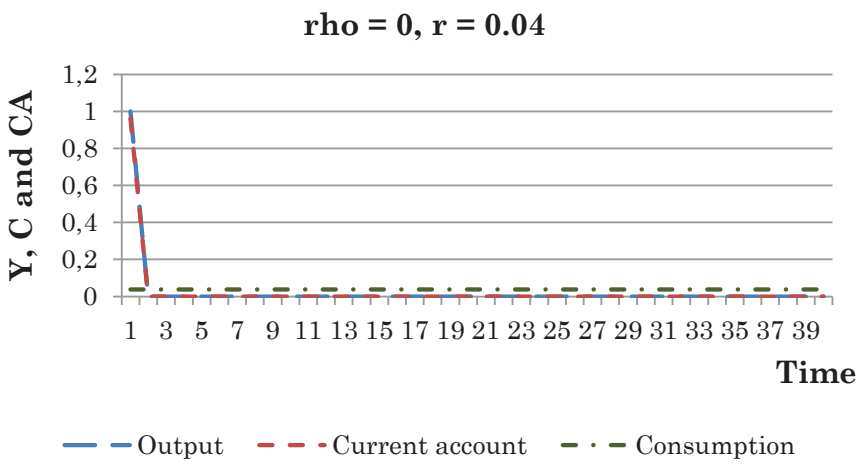
A simple simulation exercise

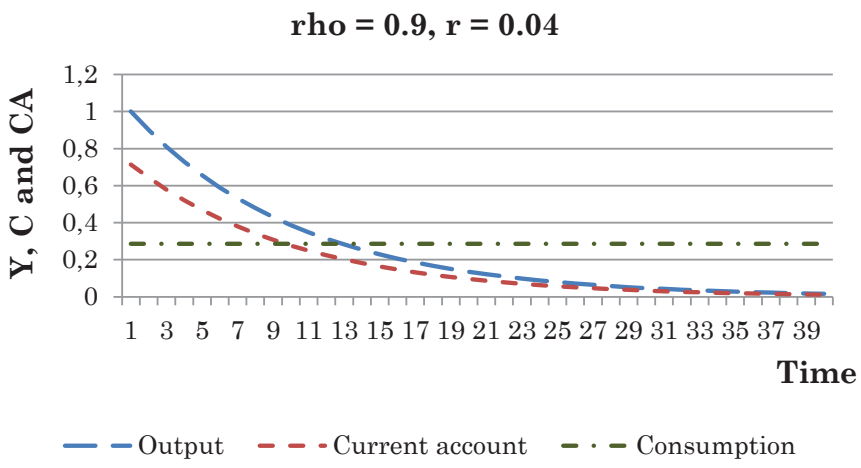
The role of ρ: Instantaneous effects on consumption and the CA

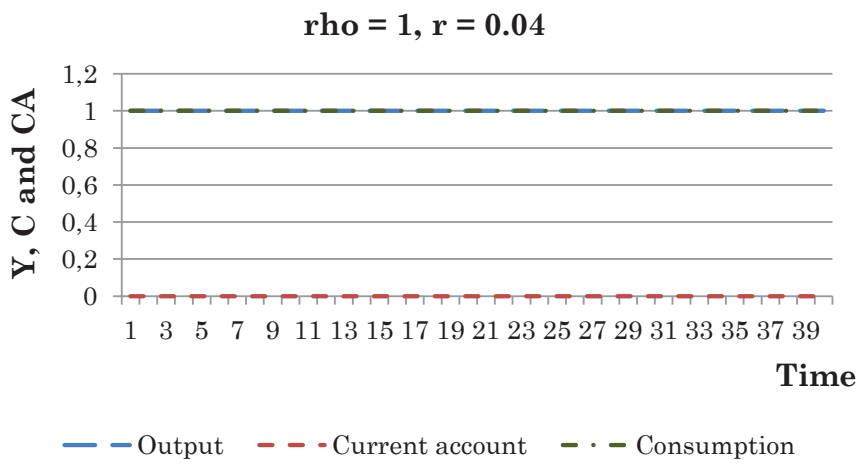


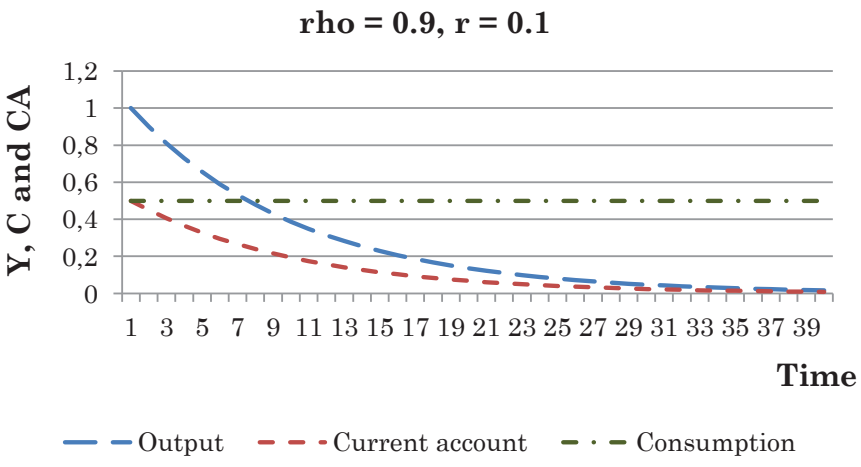
A simple simulation exercise

Impulse responses:









Lessons about the CA from the infinite horizon model

- Temporarily high output gives surpluses.
- Temporarily high investment or government spending gives deficits.
- The more persistent shocks are, the smaller is the initial effect on the current account.
- Expectations of future output growth should produce deficits.
- Deficits or surpluses can continue indefinitely (do not have to self correct), but should be stabilized by trade balances.

That's it! Please let me know if you have any questions (office B4-022).

Good luck on the exam!!