Norwegian Business School

1.B TILLEGG: MKM

BST 1612 – ANVENDT MAKROØKONOMI MODUL 5

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MINSTE KVADRATERS METODE

- Enkel regresjon - modell:

$$y_t = \beta_0 + \beta_1 x_t + u_t \tag{1}$$

- Estimat:

$$\hat{y}_t = \hat{\beta}_0 + \hat{\beta}_1 x_t \tag{2}$$

- Error:

$$\hat{u}_t \equiv y_t - \hat{y}_t \tag{3}$$

- MKM:

$$min_{\hat{\beta}_0,\hat{\beta}_1} \sum_{t=1}^{T} (\hat{u}_t)^2 = min_{\hat{\beta}_0,\hat{\beta}_1} \sum_{t=1}^{T} (y_t - \hat{y}_t)^2$$

$$min_{\hat{\beta}_0,\hat{\beta}_1} \sum_{t=1}^{T} \left(y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t\right)^2 \tag{4}$$

- (4) er problemet som løses i MKM. Førsteordensbetingelsen:

$$\frac{\partial \sum_{t=1}^{T} (\hat{u}_t)^2}{\partial \hat{\beta}_0} = 2 \sum_{t=1}^{T} \left(y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t\right)(-1) = 0$$

$$\Rightarrow \quad \sum_{t=1}^{T}\left(y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t\right) = 0 \qquad (5)$$

$$\frac{\partial \sum_{t=1}^{T}(\hat{u}_i)^2}{\partial \hat{\beta}_1} = 2\sum_{t=1}^{T}\left(y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t\right)(-x_t) = 0$$

$$\Rightarrow \quad \sum_{t=1}^{T}\left(y_t - \hat{\beta}_0 - \hat{\beta}_1 x_t\right)x_t = 0 \qquad (6)$$

- (5) og (6) kalles normalligningene. Disse brukes til å finne $\hat{\beta}_0$ og $\hat{\beta}_1$ som løser (4). Av (5):

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} \qquad (7)$$

- (7) determinerer $\hat{\beta}_0$ som løser (4). Setter derfor (7) inn i (4) og minimerer med hensyn på $\hat{\beta}_1$:

$$min_{\hat{\beta}_1} \sum_{t=1}^{T}\left(y_t - \left(\bar{y} - \hat{\beta}_1 \bar{x}\right) - \hat{\beta}_1 x_t\right)^2$$

$$min_{\hat{\beta}_1} \sum_{t=1}^{T}\left((y_t - \bar{y}) - \hat{\beta}_1 (x_t - \bar{x})\right)^2 \qquad (8)$$

$$\frac{\partial \sum_{t=1}^{T}(\hat{u}_i)^2}{\partial \hat{\beta}_1} = 2\sum_{t=1}^{T}\left((y_t - \bar{y}) - \hat{\beta}_1(x_t - \bar{x})\right)\left(-(x_t - \bar{x})\right) = 0 \qquad (9)$$

$$\Rightarrow \quad \sum_{t=1}^{T}\left((y_t - \bar{y}) - \hat{\beta}_1(x_t - \bar{x})\right)(x_t - \bar{x}) = 0 \qquad (10)$$

$$\Rightarrow \quad \sum_{t=1}^{T}(y_t - \bar{y})(x_t - \bar{x}) - \hat{\beta}_1 \sum_{t=1}^{T}(x_t - \bar{x})^2 = 0$$

$$\Rightarrow \quad \hat{\beta}_1 = \frac{\sum_{t=1}^{T}(y_t - \bar{y})(x_t - \bar{x})}{\sum_{t=1}^{T}(x_t - \bar{x})^2} \qquad (11)$$

- Oppsummert: De empiriske momentene til $\hat{\beta}_0$ og $\hat{\beta}_1$ i (2) som minimerer (4) er gitt ved (7) og (11):

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} \qquad (7)$$

$$\hat{\beta}_1 = \frac{\sum_{t=1}^{T}(y_t - \bar{y})(x_t - \bar{x})}{\sum_{t=1}^{T}(x_t - \bar{x})^2} \qquad (11)$$

- Forventningsrett og konsistens for $\hat{\beta}_1$. (11) gir:

$$\hat{\beta}_1 = \frac{\sum_{t=1}^{T}(y_t - \bar{y})(x_t - \bar{x})}{\sum_{t=1}^{T}(x_t - \bar{x})^2} \qquad (11)$$

$$\Rightarrow \quad \hat{\beta}_1 = \frac{\sum_{t=1}^{T}\big(\beta_1(x_t - \bar{x}) + (u_t - \bar{u})\big)(x_t - \bar{x})}{\sum_{t=1}^{T}(x_t - \bar{x})^2} = \beta_1 + \frac{\sum_{t=1}^{T}(u_t - \bar{u})(x_t - \bar{x})}{\sum_{t=1}^{T}(x_t - \bar{x})^2} \qquad (12)$$

- Viser forventningsrett og konsistens med utgangspunkt i (12):
  - Forventningsrett:

$$E\big(\hat{\beta}_1\big) = \beta_1 + \frac{\sum_{t=1}^{T}(E(u_t|x) - \bar{u})(x_t - \bar{x})}{\sum_{t=1}^{T}(x_t - \bar{x})^2} = \beta_1 \qquad (13)$$

  - Konsistens:

$$plim_{t\to\infty}\big(\hat{\beta}_1\big) = \beta_1 + \frac{Cov(u,x)}{Var(x)} = \beta_1 \qquad (14)$$